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COMPARISON OF PROBLEM-SOLVING PROFICIENCY OF GRADE NINE STUDENTS IN FOUR DIFFERENT MATHEMATICS PROGRAMS

A Thesis
Presented to
the Faculty of Graduate Studies
The University of Alberta

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

Department of Secondary Education

by

William T. Worbets

April 1966

ABSTRACT

The purpose of this study was to investigate the relative contributions of four Grade IX mathematics programs to the development of students' proficiency in solving mathematical problems. One group of students studied Exploring Modern Mathematics in Grades VIII and IX; a second group studied Seeing Through Mathematics in Grades VIII and IX; a third group studied Seeing Through Mathematics in Grade VIII and Exploring Modern Mathematics in Grade IX; and a fourth group studied Winston Mathematics in Grade VIII and Mathematics for Canadians in Grade IX.

The September, 1963, Cooperative School and College Ability Tests, Level Four, and Iowa Tests of Basic Skills, Form 1 (Arithmetic Problem-Solving Subtest) scores indicated that the four groups of students involved in the study were not significantly different with respect to scholastic ability and arithmetic problem-solving proficiency at the beginning of Grade VIII. In May, 1965, the Iowa Problem-Solving Subtest was readministered to the same 492 students (nineteen classes) along with a Special Problem-Solving Test prepared by the investigator. Two-way, unweighted means analysis of variance was used to test the hypotheses.

It was found that on the Iowa Problem-Solving Subtest at the end of grade nine, there were no significant differences among the group mean scores obtained by the groups of students studying the four different mathematics programs. However, on the Special Problem-Solving

Test at the end of grade nine, the Exploring Modern Mathematics students and the Seeing Through Mathematics-Exploring Modern Mathematics students scored significantly higher than either the Seeing Through Mathematics students or the Mathematics for Canadians students. The scores obtained by the Exploring Modern Mathematics students and the Seeing Through Mathematics-Exploring Modern Mathematics students did not differ significantly. Likewise, the scores obtained by the Seeing Through Mathematics students and the Mathematics for Canadians students did not differ significantly.

The findings of the study also indicate that the low ability students of the Exploring Modern Mathematics program scored significantly higher on the Special Problem-Solving Test at the end of grade nine, than the low ability students in the remaining three programs.

ACKNOWLEDGEMENTS

The writer is indebted to many people who gave generously of their time and assistance to make this study possible.

Appreciation is expressed to my advisor Dr. Geraldine Farmer for her guidance, patience, encouragement, and attention to precise expression.

The writer is further appreciative of the assistance and suggestions given by his advisory committee members; Dr. L. Doyal Nelson, Dr. S. E. Sigurdson, Mr. Wm. Coulson, and to the authorities in the field of mathematics instruction who aided in improving the Special Problem-Solving Test.

Sincere thanks are extended to the members of the Alberta Junior High School Mathematics Subcommittee, particularly Mr. J. Hrabi, who made the study possible by making arrangements with the schools and teachers participating in the experiment and by permitting me to use some of the Department of Education facilities.

Finally, the writer would like to express a special debt of gratitude to his wife, Rosalie, for her encouragement and forbearance.

Wm. T. W.

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CHAPTER I

STATEMENT OF THE PROBLEM

INTRODUCTION

The changes in mathematics caused by advances in mathematical research and the automation revolution have prompted groups of teachers, mathematicians, and psychologists to develop new mathematics programs for use at the elementary and secondary school levels. The more eminent of these groups are the School Mathematics Study Group, the University of Illinois Curriculum Study in Mathematics Project, the Ball State Teachers College Experimental Program, the University of Maryland Mathematics Project, the Developmental Project in Secondary Mathematics Southern Illinois University, and the Commission on Mathematics College Entrance Examination Board.

While each mathematics program has unique features, all programs have common elements and all have as the prime aim the improvement of mathematics instruction. Furthermore, all of these programs attempt to present new materials as a series of related topics. They stress unifying themes or ideas in Mathematics[1] such as the following:

Structure
Operations and their inverses
Measurement
Extensive use of graphical representation

---

[1] The Revolution in School Mathematics, National Council of Teachers of Mathematics, (1201 Sixteenth Street, N.W., Washington 6, D.C.), p. 22.

Systems of numeration
Properties of numbers, development of real number system
Statistical inference, probability
Sets - language and elementary theory
Logical deduction
Valid generalization.

Although each program subscribes to teaching mathematics from the point of view of unifying themes, in keeping with the philosophy of teaching mathematics in a meaningful way, there are conspicious differences in the approach to the development of problem-solving proficiency and in the approach to, and handling of, mathematical ideas. While all programs accept the premise that problem solving is the key to the teaching of mathematics, some programs emphasize the related and organized concepts of mathematics as a discipline whereas others emphasize the development of strategies by which mathematical concepts can be organized and applied to the solution of problems.

Adaptations of the various programs appearing in commercial textbooks pose a problem to the Curriculum Decision-Making Committees of the Department of Education. The Alberta Junior High School Mathematics Subcommittee deemed it necessary to assess the effectiveness of these texts in promoting understanding of mathematical ideas and in developing proficiency in problem solving. During the 1963-64 school term, teachers in a few selected Alberta schools, experimented with two different mathematics textbook series at the grade seven level. Book 1 of Exploring Modern Mathematics[2], and Book 1 of Seeing Through

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[2] M. L. Keedy, R. E. Jamison, and Patricia L. Johnson, Exploring Modern Mathematics, Book 1, (New York: Holt, Rinehart and Winston Inc., 1963).

Mathematics[3] were the texts used. In April, 1963, the Mathematics Subcommittee prepared a bulletin[4] outlining its views on the problem of mathematics curriculum reform. The bulletin was based upon reports from teachers who were experimenting with the new textbooks and upon analyses of current literature on mathematics and Junior High School Mathematics Curriculum materials. At the end of two years, Harrison[5] analyzed the effectiveness of these two mathematics programs in fulfilling the objectives expressed in the bulletin for the development of understanding of mathematical ideas. In Harrison's study the experimental group of Grade VIII students studied Book 1 and 2 of Exploring Modern Mathematics[6] or Books 1 and 2 of Seeing Through Mathematics[7], while the control group studied the currently authorized textbook, Winston Mathematics, Intermediate 2[8]. In the implications

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[3] Henry Van Engen and others, Seeing Through Mathematics, Book 1, (Chicago: Scott Foresman and Company, 1962).

[4] Alberta Junior High School Mathematics Subcommittee, Junior High School Mathematics Bulletin, April, 1963, (Mimeographed).

[5] Donald B. Harrison, "An Analyses of the Effectiveness of Three Mathematical Programs at the Grade Eight Level", (Unpublished Masters Thesis, the University of Alberta, Edmonton, 1964).

[6] M. L. Keedy, R. E. Jamison, and Patricia L. Johnson, Exploring Modern Mathematics, Book 1 and 2, (New York: Holt, Rinehart and Winston Inc., 1963).

[7] Henry Van Engen and others, Seeing Through Mathematics, Book 1 and 2, (Chicago: Scott Foresman and Company, 1962).

[8] H. L. Stein and others, Winston Mathematics, Intermediate 1 and 2, (Toronto: The John C. Winston Company, Limited, 1957).

for further research, Harrison states:

> An interesting by-product of the analysis carried out on the experimental data was the finding that thirty percent of the Winston Mathematics students answered a simple "ratio" problem in the Special Mathematics Understanding Test while only ten to thirteen percent of the Seeing Through Mathematics or Exploring Modern Mathematics students answered it correctly. Since at least one of these "modern" programs places heavy emphasis on ratios in problem solving, this phenomenon should certainly be investigated further.
>
> While a study such as the one described in this report serves the purpose of proving overall comparisons between programs, there is a need for studies that concentrate on specific aspects of the various programs being compared. For instance, a detailed study of student facility in solving mathematics problems would do much to explain the phenomenon mentioned in the preceeding paragraph as well as to point the way to improved techniques in the teaching of problem solving.[9]

In September, 1964, the Junior High School Mathematics Subcommittee of the Department of Education decided to investigate the relative contributions of the two textbook series to the development of students' problem-solving proficiency. To make such a comparison, the present study was undertaken.

PROBLEM

This investigation seeks answers to the following questions:

1. Do students studying the new textbook materials achieve equally well on a conventional mathematics problem- solving test as students studying the currently authorized textbook materials?
2. Do students studying one series of new textbook materials achieve equally well on a conventional mathematics problem-solving test as students studying another series of new textbook materials?

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[9] Harrison, op. cit., page 143.

3. Do students transferring from one series of new textbook materials to another new series achieve equally well on a conventional mathematics problem-solving test as students studying the same series of new textbook materials?

4. Do students studying the new textbook materials achieve equally well on a special mathematics problem-solving test as students studying the currently authorized textbook materials?

5. Do students studying one series of new textbook materials achieve equally well on a special problem-solving test as students studying another series of new textbook materials?

6. Do students transferring from one series of new textbook materials to another achieve equally well on a special problem-solving test as students studying the same series of new textbook materials?

NEED FOR THE STUDY

Mathematics educators define mathematical problem solving as quantitative thinking which involves the analyzing of described situations and the discovering of solution to these situations by the use of number relations. Problem solving involves the process of looking at the problem, seeking relationships, analyzing, organizing, and reorganizing these relationships until a solution becomes evident.

The Alberta Junior High School Mathematics Subcommittee in its guidelines for revised Junior High School Mathematics curriculum considered problem solving to be an important aspect of the curriculum and favoured an approach which stresses:

(a) the statement of the problem situation in the form of mathematical sentence followed by computations and then an interpretation relating the answer to the original situation.

(b) the use of a ratio or a ratio-pair approach to all problems to which it can be applied.

(c) problems involving inequalities as well as problems involving equalities.

(d) solution procedures which include graphing.[10]

Hildenbrandt in his comments on the growth of mathematical ideas also places considerable emphasis on problem solving. He states:

> ...Note that it is not merely necessary that the individual acquire a large amount of mathematical or scientific knowledge, but that the real test of his ability comes when he is confronted with a difficult problem-situation in science or further mathematics and is able to suggest or discover ways of finding an answer or the complete solution.
>
> Understanding of mathematical concepts and some skills with its techniques are necessary to both the application of mathematics in new situations and to the creation of new mathematics, but these understandings and skills are far from sufficient. To apply mathematics one must also develop proficiency in problem solving or reflective thinking....[11]

Hildenbrandt asserts that to be able to apply mathematics a learner must be interested, curious, alert to perceive interrelationships between different mathematical concepts, and able to see generalizations.

Papy emphatically claims that; "If mathematics is to be used effectively in real situations, it is not enough to have a perfect device that solves problems automatically."[12] Papy suggests that carefully chosen situations must be developed by textbooks and

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[10] Alberta Junior High School Mathematics Subcommittee, op. cit., pp. 20-21.

[11] E. H. C. Hildenbrandt, "Mathematical Modes of Thought", The Growth of Mathematical Ideas, Twenty-Fourth Yearbook, (Washington: The National Council of Teachers of Mathematics, 1959), p. 370.

[12] G. Papy, "Method and Techniques of Explaining New Mathematical Concepts in the Lower Forms of Secondary Schools", Mathematics Teacher, Vol. 58, April, 1965, pp 345-352.

teachers in order to have a creative impact on the pupils' minds. These situations should lend themselves to mathematical treatment, i.e. they must be of a type that can be conceptualized and mathematized.

There is general agreement that most of the traditional exercises in applied mathematics do not meet the criteria presented by Papy. However, is there enough or any experimental evidence to show that the new modern mathematics textbook series have developed better problem-solving situations?

Ferguson, a prominent proponent of one of the new programs being developed, expresses a similar concern.

> We are not very happy with the verbal problem situation yet, and we are still trying to find proper application problems that will aid in the teaching of the mathematics but will not detract from the mathematics because the students don't understand the physics. We are hopeful someone will make some contributions here. Professor Beberman is exploring the problem of applications in his new study. Physical problems simply do not motivate all students to learn mathematics, but for some, I am sure, it does. This is a difficult problem that I haven't seen solved in my thirty years in the business.[13]

The improvement of the mathematics curriculum prompted by the growth of mathematics itself is a never-ending process. Conflicting claims continue to be made for traditional and newer mathematics at the elementary and secondary school levels. Nevertheless, decisions must be made about the topics included, the sequence of topics, and

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[13] W. W. Ferguson, "Current Reforms in the Mathematics Curricula, A Passing Phase or Progress?", Mathematics Teacher, Vol. 57, March, 1964, pp. 143-148.

the immediate and remote goals of mathematics programs. If these are to be wise decisions, they must be based upon experimentation and research rather than on conjecture and opinion. The present study is an attempt to investigate the relative contributions of selected mathematics programs to problem solving at the Junior High School level.

HYPOTHESES

For the purpose of simplicity and clarity in the statement of the hypotheses, students studying only Exploring Modern Mathematics textbooks will be referred to as EMM; students studying only Seeing Through Mathematics textbooks will be referred to as STM; students studying both Seeing Through Mathematics and Exploring Modern Mathematics textbooks will be referred to as STM-EMM; and students studying the currently authorized textbooks will be referred to as TRAD.

The null hypotheses tested are as follows:

1. On a conventional, standardized arithmetic problem-solving test (IOWA) there are no significant differences among:
    - (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
    - (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.
2. On a Special Mathematics Problem-Solving Test there are no significant differences among:
    - (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
    - (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.

DELIMITATIONS

Although many factors may affect mathematical problem-solving proficiency, this study considered only the factors of textbook materials, scholastic ability, initial problem-solving proficiency, time devoted to the study of mathematics, and professional training and experience of teachers.

The experimental groups in this study included all Alberta Grade IX students using the new commercial mathematics textbooks series, Exploring Modern Mathematics Books 2 and 3[14] and Seeing Through Mathematics Books 2 and 3[15]. The control group was limited to six classes who were using the currently authorized textbook Mathematics for Canadians Book 1.[16]

DEFINITION OF TERMS AND ABBREVIATIONS

Alberta Junior High School Mathematics Subcommittee. A committee of mathematics teachers and educators appointed by Alberta Department of Education. The subcommittee makes recommendations regarding mathematics curricula to the Alberta Junior High School Curriculum Committee.

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[14] M.L. Keedy, R. L. Jamison, P. L. Johnson, Exploring Modern Mathematics, Books 2 and 3, (New York: Holt, Rinehart, and Winston Inc., 1964).

[15] Henry Van Engen and others, Seeing Through Mathematics, Books 2 and 3, Parts 1 and 2, (Chicago: Scott, Foresman and Co., 1964).

[16] Henry Bowers and others, Mathematics for Canadians, Book 1, (M. M. Dent and Sons (Canada) Limited and the Macmillan Company of Canada Limited, 1947).

Problem in Mathematics. A problem in mathematics is defined as a criterion to determine whether an acquired mathematical concept and number relationship are accurately applied to answer a question in a situation when directions are implicitly given.

EMM. The Exploring Modern Mathematics texts or the group of students studying these texts.

STM. The Seeing Through Mathematics texts or the group of students studying these texts.

STM-EMM. The group of students who studied STM texts while in Grades VII and VIII and EMM texts while in Grade IX.

TRAD. The currently authorized Junior High School mathematics textbooks or the group of students studying these texts.

SCAT. The Cooperative School and College Ability Test.

IOWA. The Iowa Tests of Basic Skills.

SPS Test. The Special Problem-Solving Test prepared by the investigator designed to measure mathematics problem-solving proficiency considered desirable in Alberta schools at the Grade IX level.

APS. Arithmetic Problem-Solving subtest of the IOWA Tests of Basic Skills.

Q. The quantitiative ability subtest or score of the SCAT test.

V. The verbal ability subtest or score of the SCAT test.

H. High ability group of students who obtained a total SCAT score of 96 and above.

A. Average ability group of students who obtained a total SCAT score from 85 to 95.

L. Low ability group of students who obtained a total SCAT score of 84 or less.

CHAPTER II

REVIEW OF THE LITERATURE

On reviewing the literature pertaining to the development of students' mathematics problem-solving proficiency one finds that mathematics educators place considerable emphasis upon the problem-solving process, upon factors affecting problem-solving facility, and upon instructional procedures which contribute to the development of problem-solving facility. The investigator was not able to locate any research studies or reviews of studies which attempted to compare the relative effectiveness of mathematics textbook series or programs in the development of mathematical problem-solving proficiency. Consequently, the first part of this chapter is devoted to analyses of the problem-solving process. The second part of this chapter is devoted to a review of the literature pertaining to factors affecting problem-solving facility. The final portion of the chapter is devoted to a summary of recommended procedures to be used by teachers to develop mathematics problem-solving facility, as well as the procedures recommended in the new mathematics textbook series involved in this study.

ANALYSES OF PROBLEM-SOLVING PROCESS

How we learn has been of interest to psychologists and educators since the time of Plato. John Dewey's[1] interpretation of the complete

---

[1] J. Dewey, How We Think, (Boston: D. C. Heath and Co., 1910).

art of thought (the solution of a problem) consists of five major phases: problem-presenting situations, analysis, hypotheses, deduction, and verification.

According to Dewey, the problem-presenting situation occurs when an individual is in a situation in which he is confused, or in which his previous knowledge and past learning experiences cannot give him immediate satisfaction. Thus learning (thinking) begins in a problematic situation in which the answer is desired by, but unknown to, the individual.

The next step is the analysis or examination, within the mind of an individual, of the situation in which there is dissatisfaction. He discovers why he is dissatisfied and clarifies the goal that would give satisfaction. He recognizes or states his problem.

The third element of thinking (learning) is a search for hunches, leads, tentative hypotheses. The whole process of framing and testing hypotheses until a satisfactory route to the goal has been reached is the heart of the problem-solving process.

The fourth element in Dewey's analysis of learning is deduction. By this he means the organization of the solution of the problem into a logical frame of reference. To Dewey, this was the most important phase. He says, "Information ... is not merely amassed then left in a heap; it is classified and subdivided so as to be available as it is needed,"[2] and further, "Only deduction brings out and emphasizes

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[2] Ibid., page 41.

consecutive relationships, and only when relationships are held in view does learning become more than a miscellaneous scrap-bag."[3]

The fifth step is referred to by Dewey as verification. If the learning has been accomplished, it is really to be used in new experiences. Dewey contends that if a student learned how to learn through the first four steps, it would appear that the fifth step is to a large extent the reapplication of the first four steps in a new problem with special attention to the use of the material learned. Lewis expressed this well when he said, "knowing begins and ends in experiences, but it does not end in the experience in which it begins."[4]

Dewey noted that the logical organization or structuring of knowledge is the final step (never the first step) in developing a basis for future learning. He also stressed that the steps in the complete art of thought do not necessarily occur in order one, two, three, and four. Dewey's explanation of thinking (learning) is the final structured explanation of what goes on in learning. A complete analysis cannot be made before the learner is aware of the problem. A learner has an idea of his problem, he finds further diagnosis helps him to clarify the problem, and when hypotheses fail, one after the

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[3] Ibid., page 97.

[4] C. I. Lewis, "Experience and Meanings", The Philosophical Review, Vol. 43, 1934, page 134.

other, he may return and reshape the problem. Further, the process of making a logical structure of the solution may result in the discovery of new and more fruitful hypotheses that give a better structure to the problem. There is a weaving back and forth across the whole learning situation with clarification and elimination of unnecessary details until the learner finally arrives at a mature, well ordered solution.

Thomson in The Psychology of Thinking, makes a distinction between two basic thought processes. The first is "concept formation ... which appears to take place in children up to the age of about fifteen years."[5] The second aspect of thinking is concept attainment which is "modifying and adapting one's existing concepts to new uses rather than of forming completely original concepts."[6]

Bruner made a detailed experimental study on concept attainment with adult human subjects. Subjects were shown a series of 81 cards, each with a different combination of figures, border, and colour. The subjects' task was to sort these cards in such a way as to illustrate realization of the correct principle. Bruner began with the assumption that "virtually all cognitive activity involves and is dependent on the process of categorizing."[7] According to Bruner, all interaction with

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[5] R. Thomson, The Psychology of Thinking, (Middlesex: Penquin, 1959), page 69.

[6] Thomson, loc. cit.

[7] J.S. Bruner, J.J. Goodnow, and C. A. Austin, A Study of Thinking, (New York: John Wiley, 1956), page 246.

the environment consists of dealing with classes of events, and building up a system of categories or concepts through learning. These help us to handle the stimuli which are continually encroaching upon us.

Adler in summarizing Bruner's experiments on concept attainment describes the process as follows:

> 1. A number of objects exists which exemplifies a class. These objects can be described in terms of a number of qualities or dimensions, each of which is ordered along a range of values. For example, the objects might vary with respect to colour, the possible colours being red, blue, and green.
>
> 2. For each instance encountered by the individual, a tentative prediction or decision about the nature of the concept is made.
>
> 3. Validation of the prediction comes when other environmental information arrives to confirm, invalidate, or leave indeterminate the correctness of the prediction.
>
> 4. The validation procedure (decision and test) provides information about which attributes should be noted.
>
> 5. A sequence of validations exists en route to a concept.[8]

Bruner calls this purposive pattern of behaviour a "strategy". Bruner also found that the most frequently used method of approach or "strategy" was the "focusing" technique, where the subject selects a particular attribute of the objects and explores it freely and systematically until it is proven to be wrong, before moving to another feature.

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[8] M. Adler, "Some Implications of the Theories of Jean Piaget and J. S. Bruner for Education", Research Service, The Board of Education for the City of Toronto, June, 1963, page 29.

FACTORS AFFECTING PROBLEM-SOLVING FACILITY

One of the most difficult problems facing teachers of mathematics is that of helping children to develop proficiency in solving mathematics problems. Lindstedt[9] claims this task is analogous to teaching the children how to think. In reviewing the research prior to 1944, Johnson[10] indicates that there is much conflicting evidence about the procedures that should be followed and factors that should be considered. Two things did seem apparent in his review: slow learners seem to derive some benefit from prescribed methods of attacking verbal problems; and providing students with some instruction in verbal problem solving is better than no help at all.

Upon examining the form that this instruction should take, it is interesting to note some of the recommendations made in various studies. In his investigation, Sutherland[11] stated that ability to solve problems appeared to consist of three factors of equal significance; general intelligence, a verbal factor, and a number factor. He claimed that it should be possible to attain marked improvement in the

---

[9] S. A. Lindstedt, "Changes in Patterns of Thinking Produced by A Specific Problem Solving Approach in Elementary Arithmetic," (Unpublished Doctoral Dissertation: University of Wisconsin, 1962), p. 13.

[10] H. C. Johnson, "Problem Solving in Arithmetic: A Review of the Literature", Elementary School Journal , Vol. 44, May, 1944, p. 481.

[11] J. Sutherland, "Investigation Into Some Aspects of Problem Solving", British Journal of Educational Psychology, Vol. 12, February, 1942, page 46.

proficiency of children to solve problems since number skills and verbal skills are acquired abilities.

Sutherland also found that problems set in familiar setting are easier for the child to solve. In his study, children scored thirty-six per cent higher in problems of this type compared to problems set in an unfamiliar situation, but of equal difficulty. This finding is verified in several other studies. Sutherland concluded that, "Since the object of education is to equip the child for life, and since only a very small minority will ever find themselves in an unfamiliar situation, it is reasonable to suppose we should concentrate on simple problems dealing with familiar material."[12]

It is interesting to note that upon examining the written problems appearing in arithmetic texts, Dexter[13] found that less than one per cent of all verbal problems could be classified as real in the sense that they were pertinent to a child's experience and interest.

Chase[14] made a study of factors related primarily and secondarily to the arithmetic problem-solving proficiency of 119 sixth grade children (58 girls, 61 boys). The problem-solving section of the Iowa

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[12] Ibid., page 68.

[13] C. E. Dexter, "Analysis of Written Problems in a Recent Arithmetic Series", Education, Vol. 64, April 1945, page 489.

[14] C. L. Chase, "The Position of Certain Variables in the Production of Problem-Solving in Arithmetic", Journal of Educational Research, Vol. 54, September, 1960, pp. 9-14.

Every-Pupil Test of Arithmetic was used as the criterion measure, with fifteen independent variables studied in relation thereto. The study identified only three of the fifteen independent variables as primarily related to problem-solving proficiency: arithmetic computations, reading to note details, and fundamental knowledge of arithmetic. These three factors accounted for sixty-two per cent of the variance in problem-solving scores. Since the secondary variables reported in this study demonstrated a significant association with the primary variables, Chase suggested that the secondary variables be investigated as an approach to methods of instruction which may promote competence in the primary variables, and hence, competence in solving problems.

In analyzing the causes for the lack of problem-solving proficiency, Bruckner[15] lists the following contributing factors: general intelligence, reading comprehension, computational skills, understanding of quantitative relationships, method of attack, interest, proper organization, precision and neatness, and comprehension of the problem situation.

One other factor that appears to have a bearing on the ability to solve problems is the extent of pupils' mathematical vocabulary. Many of the studies concerned with this relationship have concluded that specific training in mathematical vocabulary will result in higher achievement in problem solving. Corle says, "understanding the terms used in mathematics is a definite factor in problem-solving

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[15] L. J. Bruckner, Improving the Arithmetic Program, (New York: Appleton-Century-Crofts, 1957).

efficiency. Teachers must insure familiarity with the vocabulary of verbal problems if effective problem solving is to result."[16]

In a discussion of the nature of problem solving in arithmetic, Johnson[17] pointed out that vocabulary is one of the main factors in a student's ability to solve problems. As an example, a student can hardly be expected to solve a problem dealing with the volume of a rectangular solid unless he knows the meanings of volume and rectangular solid, and unless he knows a process for solving the problem.

Sutherland[18] emphasized the importance of vocabulary. On the basis of a recent study at the University of Illinois Teachers' College, Phillips[19] concluded that more attention should be given to ideas and the words that convey them.

The "one answer mirage" is often mentioned as another contributing factor to children's difficulty with verbal problems. Corle[20] comments:

> We have encouraged pupils to believe that there is one unique answer, and only one, to every verbal arithmetic problem. We even confirm this belief by printing the answer in the "back of the book". Perhaps the teacher delays active participation in problem solving too long, perhaps the problems are uninteresting, perhaps the vocabulary is faulty.

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[16] C. G. Corle, "Thought Processes in Grade Six Problems", The Arithmetic Teacher, Vol. 5, October, 1958, pp. 193-203.

[17] J.T. Johnson, "On the Nature of Problem Solving in Arithmetic", Journal of Educational Research, Vol. 43, October, 1949, pp. 110-115.

[18] Sutherland, op. cit., page 73.

[19] C. Phillips, "The Relationship Between Arithmetic Achievement and Vocabulary of Elementary Mathematics", The Arithmetic Teacher, Vol. 7, May, 1960, page 242.

[20] C. G. Corle, "Why Do Children Have Difficulty With Verbal Problems?", The Arithmetic Teacher, Vol. 12, January, 1965, page 13.

In 1940, Doty made a study of children's procedures in solving problems. In his conclusions he stated, "that almost the entire process of solving problems centers in the decision as to the mathematical process(es) to be used. Success at this point requires understanding, both of mathematical process and of the language which signifies need for their use."[21] In a study by Burch[22], interviews were conducted in order to determine the methods used by children in solving problems. As the children were interviewed the one factor which stood out was their lack of understanding of the effect of the fundamental processes when they applied to quantitative situations. Like Doty, Burch also came to the conclusion that children had difficulty in solving problems largely because of a lack of understanding of the meaning and the significance of the process. Pace[23] concluded that although children show great gains in problem-solving proficiency if they are merely presented many problems to solve for a period of time, they show even greater gains if systematic instruction for the purpose of developing understandings of the four fundamental processes is provided.

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[21] R. A. Doty, "A Study of Children's Procedures in the Solution of Verbal Problems", (Unpublished Doctoral Dissertation: Duke University, Durham, North Carolina, 1940), page 162.

[22] R. L. Burch, "An Evaluation of Analytic Testing in Arithmetic Problem Solving", (Unpublished Doctoral Dissertation: Duke University, Durham, North Carolina, 1949), page 119.

[23] A. Pace, "The Effect of Understanding on the Organization and Permanence of Learning, A Study in Verbal Problem Solving", (Unpublished Doctoral Dissertation: Syracuse University, Syracuse, New York, 1959), page 113.

A study made by Alexander[24] at the Grade seven level tried to answer the following questions. Are there characteristic differences between high and low achievers in seventh grade problem solving? Do they differ significantly in various types of reading skills? What quantitative skills are necessary for successful achievement in problem solving? Are any specific mental abilities closely associated with problem solving? Does socio-economic status influence the seventh grader's success in problem solving? Pairs of high and low achievers in seventh grade problem solving matched according to sex, I.Q., and mental age in months were compared with respect to the following factors and abilities: specific mental abilities, quantitative skills, general reading skills, problem solving reading skills, interpretation of quantitative materials, and socio-economic status.

General conclusions derived from the study were as follows: Two specific abilities are essential to the seventh grader's ability to solve mathematics problems. His ability to solve logical problems through reasoning and his ability to understand words as verbal concepts are strongly associated with his ability to solve mathematics problems. The problem-solving proficiency of seventh graders seems to depend in large measure upon their ability to (1) interpret and use the number system, (2) employ the fundamental processes with various types of numbers, and (3) comprehend and use the concept of measurement. To achieve in problem solving the seventh graders should be able to (1) understand the theme of a reading selection, (2) understand the

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[24] V. E. Alexander, "Seventh Grader's Ability to Solve Problems", School Science and Mathematics, Vol. 60, 1960, pp. 603-606.

main idea of a paragraph, (3) infer logical ideas which are implied, (4) grasp small ideas in written materials, and (5) understand the meaning of words in context. Problem solving is synonymous with the ability to visualize and interpret quantitative facts and relationships.

RECOMMENDED PROCEDURES TO DEVELOP PROBLEM-SOLVING FACILITY

Many educators contend that our knowledge about any subject consists of information and of "know-how"*. There is no doubt in their minds that "know-how" is much more important than mere possession of information. Therefore at any grade level the schools should impart, along with a certain amount of information, a certain degree of "know-how" to the students.

Polya defines "know-how" in mathematics as:

> The ability to solve problems -- not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity. Therefore, the first and foremost duty of the high school in teaching mathematics is to emphasize methodical work in problem solving.[25]

He is convinced that this mathematical "know-how" can and should be developed through imitation and practice by students of patterns or models. Polya recommends the use of the heuristic approach by means of questions to any problem, the first question being: "What kind of problem is this?".[26] He characterizes all problems into two general types, problems "to find" and problems "to prove". "What kind of

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*Polya's term.

[25] G. Polya, Mathematical Discovery Volume 1, (New York: John Wiley & Sons, Inc., 1962), page viii.

[26] Ibid., page 118.

problem is this", leads the student to the next question, "What can be done about this kind of problem?", which may suggest the type of solution or model a student may follow. He is also convinced that the two types of problems can be solved by any one or several of the following patterns; the pattern of two loci, the pattern of similar figures, the pattern of auxiliary figures, the Cartisian pattern, recursion, and superposition.

In plain geometry, Polya suggests resorting to the patterns of two loci, similar figures and auxiliary figures. He defines the pattern of two loci as:

> First, reduce the problem to the construction of ONE point. Then, split the condition into TWO parts so that each part yields a locus for the unknown point; each locus must be either a straight line or a circle.[27]

and the pattern of similar figures as:

> If you cannot construct the required figure, think of the possibility of constructing a figure SIMILAR to the required figure.[28]

The pattern of auxiliary figures is described by Polya in the following manner:

> Try to discover some part of the figure or some closely RELATED FIGURE which you can construct and which you can use as a stepping stone in constructing the original figure.[29]

The Cartisian pattern follows the scheme that Descartes

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[27] Ibid., page 5.

[28] Ibid., page 9.

[29] Ibid., page 15.

expected to be applicable to all types of problems:

> First reduce any kind of problem to a mathematical problem. Second, reduce any kind of mathematical problem to a problem of algebra. Third, reduce any problem of algebra to the solution of a single equation.[30]

The pattern of recursion is defined as the finding of, "the terms one after the other, successively, recursively, by going back or recurring each time to the foregoing terms,"[31] and the pattern of superposition is outlined as, "starting from a leading special situation we attain the general solution by superposition of particular cases."[32]

Furthermore, Polya suggests that students should be continually guided to reflect on various procedures after the solution has been obtained. During this reflection they should ask themselves, where could I use this problem? Is there some trick worth learning, that I could use the next time in a similar situation? What could I have done better? And there are many others. Polya concludes, "but the best question is the one that comes spontaneously to mind".[33]

Bruner concurs with this approach when he outlines the benefits derived from the experience of learning through discovery:

> ... It is my hunch that it is only through the exericse of problem solving and the effort of discovery that one

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[30] Ibid., page 22.

[31] Ibid., page 67.

[32] Ibid., page 108.

[33] Ibid., page 210.

> learns the working heuristics of discovery; the more one has practice,, the more likely one is to generalize what one has learned into a style of problem solving or inquiry that serves for any kind of task encountered--or almost any kind of task.[34]

The literature also contains numerous articles warning against the trend towards excessive rigor, formalism and generalization found in some of the new textbooks. Although no specific charges are made against any specific sets of materials, it is understandable that an overly formalistic "modern" program could be just as undesirable as any traditional program with respect to rote learning. Recently, Irish[35] conducted a research study to determine whether children's ability to solve verbal problems in mathematics could be improved (1) by developing their ability to generalize the meanings of the number operations and relationships among these operations and (2) by developing an ability to formulate original statements to express these generalizations as they are attained. This experiment was carried out at the Grade six level in Santa Barbara, California, as a result of a testing program which revealed that many students whose scores fell above the fiftieth percentile in computation ranged in the tenth percentile in verbal problem solving. It was hypothesized that the major difficulties were of two kind: (1) children had insufficient general understanding of the nature of the number operations and the relationships within the number system, although they could perform the

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[34] J. S. Bruner, On Knowing: Essays for the Left Hand, "The Act of Discovery", page 89.

[35] E. H. Irish, "Improving Problem Solving by Improving Verbal Generalization", The Arithmetic Teacher, Vol. 11, March, 1964, pp. 160-175.

operations correctly, (2) the children could not move comfortably in their thinking from the number symbols to the verbal language of daily use in which the problems are stated.

It was assumed that an approach through verbal statement would be a reasonable way to attack both difficulties since continued practice in using ordinary language to explain the number operations in general terms should deepen understanding. Since repeated attempts to achieve a precise verbal description should lead to a clearly defined pattern that would be understood in the same language as the verbal problems, Irish further assumed that children's improvement in the proficiency to solve verbal problems would occur through work in the quality of their generalization related to computation.

Data collected showed that for two consecutive years children in the experimental group made significantly greater average growth in problem solving and computation than children in the control group. This occurred when instruction in computation was modified to include emphasis upon children's growth in ability to formulate verbal generalizations appropriate to the topic under study. Irish concluded "... that such instruction should be included as a teaching procedure for problem solving to ensure that children made functional use of their computational ability in problem solving."[36]

In recent years increasing attention has been directed toward the place of mental arithmetic in developing problem-solving proficiency in the elementary and intermediate school. Gane states:

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[36] Ibid., page 175.

> The fact that daily living frequently requires a person to compute without the aid of pencil and paper suggests that our instructional practices in this area should be reappraised.[37]

Flourney's[38] study showed the effects of a 52-day program on 550 pupils in the intermediate grades. The children were given a ten to twelve minute lesson each day of the experiment. Not only did their ability to compute and solve problems without pencil and paper improve significantly, but their progress in written arithmetic was also highly satisfactory. In addition, their growth in arithmetic achievement as measured by standardized tests showed a gain of eight months in the fifty-two-day period. Similar results were obtained with teacher-made written tests.

Gane emphasized that provision for practice in this area should be provided at various levels since range of individual differences in this subject is considerable. When Hall[39] presented fifty carefully selected mental arithmetic problems to 179 intermediate grade pupils, their scores ranged from two to forty-seven. He found that the correlation between achievement in this field and general intelligence was 0.74 and concluded that there are definite limits in the ability of dull children to solve complicated two-and-three-step problems without pencil and paper.

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[37] James T. Gane, "Research Should Guide Us", The Arithmetic Teacher, Vol. 9, December, 1962, pp. 441-445.

[38] Mary Francis Flourney, "The Effectiveness of Instruction in Mental Arithmetic", Elementary School Journal, Vol. 55, November, 1954, p. 52.

[39] Jack V. Hall, "Solving Verbal Arithmetic Problems Without Pencil and Paper", Elementary School Journal, Vol. 48, December 1947, p. 235.

Gane claims that:

> A regular and carefully planned program of mental arithmetic provides a realistic preparation for the everyday use of arithmetic which the child encounters out of school. Improvement in mental arithmetic tends to improve ability in all phases of arithmetic.[40]

In a study made by Stevenson[41], a group of 1,014 fifth, sixth, and seventh grade children received instruction in solving problems for three fifteen-minute periods a week for twelve weeks. The first three weeks were devoted to the reading and analysis of problems. The children were drilled in finding the facts pertaining to the problem, in deciding upon the processes to be used in solving the problem, and in finding the answer in round number. During the second three week period, the children were given practice in solving a large variety of problems from actual life situations. During the third three week period, problems without numbers were solved. Then for three weeks, the different words in problems were studied. Stevenson found that these methods of instruction brought about a material improvement in ability to solve problems as measured by the Buckingham Scale for problems in arithmetic and the Stevenson Problem Analysis Test. No attempt was made to determine the relative value of the different methods, nor was any attempt made to determine the effectiveness of these methods as compared to other methods.

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[40] Gane, op. cit., pp. 441-445

[41] R. P. Stevenson, "Increasing the Ability of Pupils to Solve Arithmetic Problems", Educational Research Bulletin, Vol. 10, pp. 276-270.

Pace[42] conducted a study in which she was directly concerned with the importance of an understanding of the four fundamental processes in arithmetic and its relation to the successful solving of verbal problems. In her conclusions she points out that teachers need to see that children are given many varied experiences in solving problems, children should be allowed to solve problems in various ways and lastly:

> ... the teacher should provide for the development of understandings of the four fundamental processes since this is a vital factor as far as improvement in problem solving is concerned.
>
> Although children show great gains in problem solving ability if they are merely presented with many problems to solve for a period of time, they show even greater gains if systematic instruction for the purpose of developing understandings of the four fundamental processes is provided.[43]

Procedures Used by the EMM Texts

Proofs and applied problems are begun in a most informal way in Book 1.[44] Towards the end of Book 1 and beginning of Book 2,[45] the materials force the students to think more carefully about applied

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[42] Angela Pace, "The Effect of Understanding on the Reorganization and Permanence of Learning, A Study in Verbal Problem Solving", (Unpublished Doctoral Dissertation: Syracuse University, Syracuse, New York, 1959).

[43] Ibid., page 113.

[44] M. L. Keedy, R. L. Jameson, P. L. Johnson, N. M. Purvis, T. P. Atkinson, Exploring Modern Mathematics, Book 1, (Toronto: Holt, Rinehart, and Winston, Inc., 1965)

[45] M. L. Keedy, R. L. Jameson, P. L. Johnson, N. M. Purvis, T. P. Atkinson, Exploring Modern Mathematics, Book 2, (Toronto: Holt, Rinehart, and Winston, Inc., 1965).

problems and provide experience with more sophisticated ideas about proofs. At the end of Book 2, and beginning of Book 3[46] a general strategy for solving problems is developed. This strategy used throughout the text, consists of the following four steps:

1. Study and understand the situation.
2. Describe the situation in mathematical language.
3. Re-describe the situation in more useful mathematical language.
4. From the re-description, get the answer to the problem.[47]

Various problems are presented by types making it convenient for students' reference and gaining experience, but it is strongly recommended that students be trained to use their own imaginations in describing problem situations mathematically. Based on the assumption that the most difficult step in problem solving is translating the problem into mathematical language, substantial amount of practice in translating, and only translating (without solving) is provided.

The following example illustrates the strategy recommended and the division of problems into types:

> You have now seen the basic principle by which we can solve work problems. It is this:
>
> If a job can be completed in n hours (or days), then $\frac{1}{n}$ of it can be completed in 1 hour (or day).
>
> For example, if a man can dig a trench in 6 hours, then he can do $\frac{1}{6}$ of it in one hour. Now let's work a problem using this principle.

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[46] M. L. Keedy, R. L. Jameson, P. L. Johnson, Exploring Modern Mathematics, Book 3, Teacher's Edition, (New York: Holt, Rinehart, and Winston, Inc., 1964).

[47] Ibid., page 3.

Example:

In a factory there are two pipes that carry water to fill a tank. If one pipe is used alone it takes 12 hours to fill the tank. If both pipes are used together it takes 5 hours. How long would it take to fill the tank using the second pipe alone?

Step 1. Study and understand the situation.
First pipe alone -- 12 hours.
Both pipes together -- 5 hours.
Find -- Time required for second pipe alone.

Step 2. Describe the situation mathematically.

Together they fill $\frac{1}{5}$ of the tank in one hour.

The first pipe fills $\frac{1}{12}$ of it in 1 hour.

Suppose the second pipe can fill the tank in n hours.

Then it fills $\frac{1}{n}$ of it in one hour.

So in one hour both pipes fill $\frac{1}{12} + \frac{1}{n}$ of the tank.

They also fill $\frac{1}{5}$ of it. We have an equation.

$$\frac{1}{12} + \frac{1}{n} = \frac{1}{5}$$

Step 3. Re-describe the situation.
We solve the equation:

$$\frac{1}{12} + \frac{1}{n} = \frac{1}{5}$$

$$\frac{1}{n} = \frac{1}{5} - \frac{1}{12} \quad (\text{adding } -\frac{1}{12})$$

$$= \frac{1}{5} \cdot \frac{12}{12} - \frac{1}{12} \cdot \frac{5}{5} = \frac{12}{60} - \frac{5}{60}$$

$$\frac{1}{n} = \frac{7}{60}$$

$$n = \frac{60}{7} \text{ and } n \neq 0$$

The solution set is $\frac{60}{7}$.

Step 4. From the re-description, we get the answer to the problem. We check and notice that together the pipes fill $\frac{1}{12} + \frac{1}{\frac{60}{7}}$ or $\frac{1}{5}$ of the tank in one hour. If we

> multiply by 5 we get 1, which means all of the tank is filled.
>
> The answer is $8\frac{4}{7}$ hours.[48]

The authors of the EMM series stress that teachers should carefully note certain guidelines when applied problems are studied or solved. The guidelines are:

1. Allow time for the student to think. Make it a regular practice not to collect a problem until the student has had several days to work on it, depending on student ability and difficulty of the problem.

2. Do not assign more than one or two of these problems at a time. It is better to include one problem or so at regular intervals than to use several consecutive periods on applied problems. These problems are placed together in sets, but not because they should all be used at once.

3. Try to assign problems of appropriate difficulty. Hints for you are given in this book, so that you won't have to work the problem to find out how hard it is.

4. If students are capable of it, avoid studying applied problems by type. It is better if bright students can learn how to set up problems by themselves. In any event, avoid giving rules for solving problems of a given type. Hints may be needed but rules as such tend to defeat the purpose of this kind of exercise. The main difficulty is finding how to make a mathematical description of a problem situation. If one has a rule, then he does not get the right kind of practice in this respect.[49]

Procedures Used by STM Texts

STM stresses the importance of pure mathematics in order to relate the development of mathematical ideas in their own right, but

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[48] Ibid., page 47.

[49] Ibid., page xvii.

does not neglect the importance of practical problem solving. Through applied problems, mathematics is related to situations in the physical world. As new types of conditions are introduced, these conditions are applied to problem solving. The series uses a variety of situations from science, geography, business, and other fields.

The STM problem-solving approach is defined as follows:

> ... The students are taught first to translate the structure of a problem situation into a mathematical condition. They then write a mathematical sentence that expresses a condition for the problem. They find the solution set of the condition and use this condition set to get an answer (or answers) for the problem.[50]

The authors claim that this approach provides cohesion because it brings together ideas on sets, conditions, variables, connectives, set operations, and solution sets. Furthermore, this approach makes it possible to unify and simplify problems concerning ratio and comparisons, such as problems involving per cent and problems that in traditional texts are known as "ratio and proportion" problems.

The program is not organized around social arithmetic but social arithmetic is included whenever it is appropriate in terms of the mathematical ideas that are being developed.

The authors also make clearly evident that there are two major themes in the STM program.

> (1) understanding of mathematical ideas (not memorization of number facts); and (2) integration of mathematical ideas (not compartmentalization).[51]

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[50] Henry Van Engen and others, Seeing Through Mathematics, Teaching Guide, Book 1, Special Edition, (Toronto: W. J. Gage Ltd.,) page 16.

[51] *Ibid.*, page 17.

The following example illustrates the strategy recommended by the series. It is built around the idea of "guided discovery".

Next you will study problems that involve compound conditions in two and three variables.

Read the problem: Evelyn has enough picture-frame molding to make a rectangular picture frame with a perimeter of 59 in. She wants the length of the frame to be 3 times the width of the frame. What should be the length and the width of the frame in inches?

A Use y as a variable for the length of the frame and use x as a variable for the width of the frame. Explain how the condition expressed at the right can be obtained from the formula for the perimeter of a rectangle. $59 = 2y + 2x.$

You know that the length of the frame is to be 3 times the width. To express this idea, you can develop a condition for direct variation. Remember that y is a variable for the length of the frame and x is a variable for the width. Since the length of the frame is to be 3 times the width, you can use the condition expressed at the right as another condition for the problem. $y = 3x.$ The condition $y = 3x$ tells you that the length varies directly as the width varies and that 3 is the constant of variation.

The sentence below expresses a compound condition for the problem.

$59 = 2x + 2y \wedge y = 3x.$

B Look at the following:

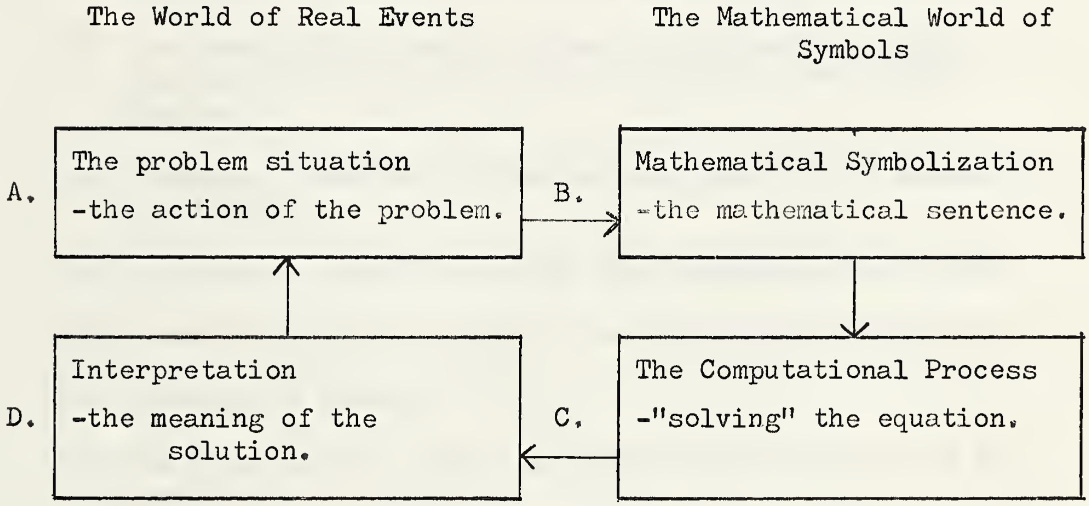
A $59 = 2y + 2x \wedge y = 3x.$
B $59 = 2(3x) + 2x \wedge y = 3x.$
C $59 = 6x + 2x \wedge y = 3x.$
D $59 = 8x \wedge y = 3x.$
E $7\frac{3}{8} = x \wedge y = 3x.$
F $7\frac{3}{8} = x \wedge y = 22\frac{1}{8}.$

Condition A is a compound condition for the problem. Explain the steps that were used to obtain condition F from condition A.

C Tabulate $\{(x, y) \mid 59 = 2y + 2x \wedge y = 3x.\}$

D Which component of $(7\frac{3}{8}, 22\frac{1}{8})$ is the width of the picture frame in inches? Give the answer to the problem.[52]

Lindstedt[53] calls this portion of thinking a structure for problem solving. He visualizes the structure as a square, the left hand side is found in the world of physical events and the right hand side is found in the abstract mathematical world. Lindstedt's conceptual model[54] of the structure appears below:


The World of Real Events
The Mathematical World of Symbols
A.
The problem situation
-the action of the problem.
B.
Mathematical Symbolization
-the mathematical sentence.
The Computational Process
-"solving" the equation.
C.
D.
Interpretation
-the meaning of the solution.

In their approach to problem solving the EMM and STM textbooks emphasize student understanding of the structure of the problem and the description of the situation in the form of a mathematical sentence. Furthermore, both series stress the importance of precision of expression

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[52] Henry Van Engen and others, Seeing Through Mathematics, Book 3, Part 1, (Chicago: Scott, Foresman and Co., 1964), pp. 109-110.

[53] S. A. Lindstedt, The Problem of Solving Arithmetic Problems, (Toronto: W. J. Gage Ltd., 1965).

[54] Ibid., page 26.

when making generalizations. However, the STM materials seem to be more rigidly committed to precise mathematical presentation than the EMM materials. In contrast, the currently authorized textbooks seem to place emphasis on manipulative and procedural techniques to obtain the correct answer. Throughout the textbooks steps for attacking different verbal problems are implicitly enumerated and explained. The following suggested plan of attack illustrates the order of the steps:

> (1) Read the problem carefully to find out what is needed. Sometimes a diagram is helpful in picturing the facts.
>
> (2) Let some letter, say x, stand for the unknown number, or for the measure of the unknown quantity, as the case may be.
>
> (3) Using x, translate the English statements into algebraic expressions.
>
> (4) Re-read the problem to find what expressions are equal.
>
> (5) Form an equation from the equal algebraic expressions.
>
> (6) Solve this equation for x.
>
> (7) Test whether the answer satisfies the conditions of the problem.
>
> Each problem of the type you will meet in algebra this year involves the finding of an unknown number from an equation. The suggested seven steps may be helpful in arriving at a solution. You will do well to think about the following comments on these steps.[55]

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[55] Henry Bowers and others, Mathematics for Canadians, Book 1, (J. M. Dent and Sons (Canada) Limited and the Macmillan Company of Canada Limited, 1947).

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

The purpose of this study is to provide information that may help to answer the following questions. Do students studying the new textbook materials in Grades 8 and 9 achieve equally well at the end of Grade 9 on a conventional mathematics problem-solving test as students studying the currently authorized textbooks in Grades 8 and 9? Do students studying one series of new textbook materials in Grades 8 and 9 achieve equally well at the end of Grade 9 on a conventional mathematics problem-solving test as students studying another series of new textbook materials in Grades 8 and 9? Do students transferring from one series of new textbook materials at the end of Grade 8 to another series of new textbook materials in Grade 9 achieve equally well at the end of Grade 9 on a conventional mathematics problem-solving test as students studying only one series of new textbook materials in Grades 8 and 9? Do students studying the new textbook materials in Grades 8 and 9 achieve equally well at the end of Grade 9 on a special mathematics problem-solving test as students studying the currently authorized textbooks in Grades 8 and 9? Do students studying one series of new textbook materials in Grades 8 and 9 achieve equally well at the end of Grade 9 on a special mathematics problem-solving test as students studying another series of new textbook materials in Grades 8 and 9? Do students transferring from one series of new textbook materials at the end of Grade 8 to another

series of new textbook materials in Grade 9 achieve equally well at the end of Grade 9 on a special mathematics problem-solving test as students studying only one series of new textbook materials in Grades 8 and 9?

Subjects Included in the Study

Nineteen Grade 9 mathematics classes, including fourteen urban and five non-urban, participated in the experiment during the 1964-65 school term. Of the nineteen classes included in the study, four, referred to as EMM, studied Exploring Modern Mathematics, Book 3; two, referred to as STM-EMM, also studied Exploring Modern Mathematics, Book 3; seven, referred to as STM, studied Seeing Through Mathematics, Book 3; six, referred to as TRAD, studied the traditional Mathematics for Canadians, Book 1. Included in the study were 492 students, five EMM teachers, six STM teachers, and six TRAD teachers. Table I shows the distribution of students and teachers according to type of mathematics programs studied.

TABLE I

NUMBER OF CLASSES, STUDENTS, AND TEACHERS PARTICIPATING IN THE FOUR MATHEMATICS PROGRAMS

Programs	Number of Classes	Number of Students	Number of Teachers
EMM	4	104	3
STM-EMM	2	52	2
STM	7	197	6
TRAD	6	139	6
Total	19	492	17

All the students included in this study had participated in the investigation conducted by Harrison[1] during the previous year, 1963-64. With the exception of the STM-EMM group, all students continued in Grade 9 the same type of mathematics programs they had taken in the 1963-64 school term. Furthermore, as many teachers as possible were assigned the same type of mathematics programs they had taught the previous year. Only four of the seventeen teachers included in this study did not participate in the experiment during the 1963-64 school term. It was necessary to add only one new teacher to each of the four mathematics programs because of staff changes.

Background of Students and Teachers

At the end of the experiment, teachers involved in the experiment were requested to complete a questionnaire. This questionnaire provided information about teachers' academic preparation and experience, mathematics instruction time by classes, and proportions of the total EMM, STM, and STM-EMM textbook series that each class covered by the end of Grade IX.

In Table II each column represents a single teacher in the respective instructional program. The numbers in the columns opposite the description of the background indicate the activities the teacher participated in, and the number of university courses the teacher took. The data reveals that the EMM, STM, and STM-EMM teachers participated

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[1] Donald B. Harrison, "An Analyses of the Effectiveness of Three Mathematical Programs at the Grade Eight Level", (Unpublished Master's Thesis, The University of Alberta, Edmonton, 1964).

TABLE II

MATHEMATICS BACKGROUND OF TEACHERS TEACHING THE EXPERIMENTAL AND CONTROL CLASSES

Background Description	EMM Teachers			STM Teachers						STM-EMM Teachers		TRAD Teachers					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Number of inservice mathematics courses or programs attended	2	2	2	2	2	2	5	2	0	2	2	0	0	1	1	1	1
Number of University mathematics courses taken that interpret modern mathematics	2	2	1	0	0	2	1	0	3	2	0	0	1	1	1	3	2
Number of other University mathematics courses taken	2	2	0	0	0	3	0	1	1	3	1	7	2	2	3	2	5
Total	6	6	3	2	2	7	6	3	4	7	3	7	3	4	4	6	8
Average	5			4						5		5.3					

in more inservice programs than the TRAD teachers. However, the TRAD teachers took more conventional university mathematics courses. The four groups of teachers do not appear to be different with respect to mathematical background.

The averages in Table III show that the TRAD teachers have had more teaching experience in Grade IX mathematics but the four groups of teachers have taught a similar number of years.

TABLE III

AVERAGE TEACHING EXPERIENCE FOR EACH GROUP OF TEACHERS

Experience Description	EMM	STM	STM-EMM	TRAD
Average number of years of teaching experience	12	13	14	13
Average number of years each teacher taught Grade IX mathematics	6	6	5	9

Table IV indicates that the average classroom instruction time did not vary appreciably.

TABLE IV

AVERAGE CLASSROOM INSTRUCTION TIME FOR EACH GROUP OF STUDENTS

Time Description	EMM	STM	STM-EMM	TRAD
Average daily period length in minutes	41	41	43	43
Average weekly instruction time in minutes	224	233	236	231

Tables V, VI, and VII show the proportions of the total Junior High School EMM, STM-EMM, and STM textbook series that each class covered by the end of Grade IX. The proportions were determined by dividing the number of chapters or units studied in the particular series by the total number of chapters or units in the three-year sequence of the particular series. To obtain the proportions for the STM-EMM classes, units of the first two-year sequence of the STM series and chapters of the third-year EMM textbook were used. The EMM classes are numbered E_1 to E_4, STM classes S_1 to S_7, and STM-EMM classes SE_1 and SE_2.

The average textbook series coverage for the EMM and STM-EMM classes is approximately eighty-five per cent and for the STM classes is approximately eighty per cent. The TRAD classes had completed the recommended chapters of the prescribed text.

TABLE V

PROPORTION OF TEXTBOOK SERIES COMPLETED BY EACH EMM CLASS

Class Number	Total Chapters in series	Total Chapters completed	Proportion Completed
E_1	28	25	.89
E_2	28	25+	.89+
E_3	28	24+	.86+
E_4	28	24+	.86+

Averrage proportion completed = .87+

TABLE VI

PROPORTION OF TEXTBOOK SERIES COMPLETED BY EACH STM-EMM CLASS

Class Number	Total Units and Chapters in series	Total Units and Chapters Completed	Proportion Completed
SE_1	26	21+	.81+
SE_2	26	23+	.88+

Average proportion completed = .84+

TABLE VII

PROPORTION OF TEXTBOOK SERIES COMPLETED BY EACH STM CLASS

Class Number	Total Units in Series	Total Units Completed	Proportion Completed
S_1	25	16	.64
S_2	25	16	.64
S_3	25	20	.80
S_4	25	20	.80
S_5	25	22	.88
S_6	25	22	.88
S_7	25	21	.84

Average proportion completed = .78

Development and Description of the Special Problem-Solving Test

Since the primary purpose of this study was to investigate the relative contributions of the four mathematics programs to problem-solving at the Junior High School level, and since there was no published test for measuring this proficiency, a special test was developed. The multipe-choice Special Problem-Solving Test, referred to a SPS Test, emphasized problems involving algebraic concepts and procedures because the three textbooks are too diversified in the study of geometry. In fact, the traditional text delays extensive study of geometric relations until the latter part of the course. The items of the Special Problem-Solving (SPS) Test were divided into the following algebraic categories:

(a) linear relations

(b) proportional relations

(c) quadratic relations.

The final form of the thirty-item test was to consist of twelve items dealing with linear relations, twelve dealing with proportional relations, and six dealing with quadratic relations.

It was considered important that the test items should be equally understandable to all groups of students taking part in the experiment. In designing the test items special terminology and symbols were omitted because "traditional" students studying _Mathematics for Canadians, Book 1_, would not be familiar with much of the special verbalism and notations, and even the two modern texts differ in form of symbolism. The investigator was careful to include in the test only

those items which contained physical situations and mathematics principles that the four groups of students would have studied. Each test question was carefully cross-checked with the three textbook series to insure that it was closely related or parallel to the mathematical experiences provided by the texts. Furthermore, the questions were selected and structured to include the ratio, comparison, additive, subtractive, multiplicative, and divisive problem situations and their variants within the real number system.

It is generally accepted that the most difficult and important step of the problem-solving strategy is the expression of the physical condition in the form of a mathematical sentence. Accordingly, the majority of the questions were designed to test this aspect. The Special Problem-Solving Test minimized the computational skills because the IOWA test which is computationally oriented could provide this information.

To insure that the physical situations and mathematical concepts involved were compatible with those currently expected of Grade IX students, ideas for the SPS test questions were collected mainly from the text materials published by the School Mathematics Study Group (SMSG)[2] and University of Illinois Committee on School Mathematics (UICSM)[3]. Ideas for some of the items came from the 26th Yearbook

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[2] School Mathematics Study Group, Mathematics for Junior High School, Volume 2, (Parts 1 & 2), and First Course in Algebra, (Parts 1 & 2), (New Haven and London: Yale University Press, 1961).

[3] University of Illinois Committee on School Mathematics, Unit 1, The Arithmetic of Real Numbers, Unit 2, Generalizations and Algebraic Manipulations, and Unit 3, Equations and Inequations, (UICSM, Urbana, Illinois, 1960).

of the National Council of Teachers of Mathematics (NCTM)[4] and various other sources mentioned in the related literature.

Early in the development of the test, the investigator prepared fifty problem-solving test items. Care was exercised to keep the language precise but neutral so that it would be meaningful to all the students, regardless of the text studied. The first draft of the Special Problem-Solving Test consisting of fifty four-alternative multiple choice questions was duplicated and circulated to members of the Alberta Junior High School Mathematics Subcommittee, and to staff members of the Faculty of Education and the Mathematics Department of the University of Alberta. On the basis of the critical appraisals, many of the items were strengthened by rewording the stem or distractors. Some questions were omitted and new items were added.

The Special Problem-Solving Test consisting of sixty-three items (twenty-eight dealing with linear relations, twenty-three dealing with proportional relations, and twelve dealing with quadratic relations) without answer alternatives was mimeographed and administered to forty-seven Alberta Grade X students. The purpose of administering this test was to obtain pupil responses which would enable the investigator to validate or change stems or multiple-choice distractors of the test items.

A second draft of 59 items called the pilot study version of the test was prepared. This version consisted of three sections: Section A contained twenty-six linear relation items, Section B contained twenty-

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[4] May A. Sobel and Donovan A. Johnson, "Analysis of Illustrative Items", Evaluation of Mathematics, Twenty-sixth Yearbook, (Washington: The National Council of Teachers of Mathematics, 1961), pp. 71-91.

two proportional relation items, and Section C contained eleven quadratic relation items. The test was administered in two parts to ninety-three Grade IX students in three heterogeneous classes. Students included in the pilot study were not included in the main experiment. A class located in an Alberta village had studied Mathematics for Canadians, Book 1. A class in Vancouver, British Columbia, had studied Modern Elementary Algebra[5] in Grade lX, while the third class in Saskatoon, Saskatchewan had studied Modern Algebra[6] in Grade IX. The two latter classes had studied modern mathematics programs for at least one year prior to Grade IX.

From the test data the mean, standard deviation, and Kuder-Richardson Formula 20 reliability index[7] were determined. The difficulty index, and biserial correlation for each test item were also calculated. The calculated KR-20 reliability index was .8375. The Kuder-Richardson Formula 20 is a measure of the internal consistency, or homogenity, or scalability, of the test material. If the test items have high intercorrelations, then the reliability coefficient will be high. If the intercorrelations are low because the items measure different attributes, then the reliability coefficient will be low.

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[5] Eugene D. Nichols, Wagner G. Collins, Modern Elementary Algebra, (New York: Holt, Rinehart, and Winston, Inc., 1961).

[6] Mary P. Dolciani, Simon L. Berman, Julius Freilich, Modern Algebra, Book 1, (Boston: Houghton Mifflin Company, 1962).

[7] George A. Ferguson, Statistical Analysis in Psychology and Education, (New York: McGraw-Hill Book Company, Inc., 1959), pp. 280-281.

The procedure used to calculate the biserial correlation is also outlined by Ferguson.[8] Generally, the higher the biserial correlation of an item, the better is its power of discrimination. The difficulty of an item, calculated as the percentage of the students who answered the items correctly, also has some bearing on the choice of items where two or more have similar powers of discrimination. Based on Helmstadter[9] and recommendations of personnel from the Educational Psychology Department, Faculty of Education, University of Alberta, the investigator selected items within a difficulty index of .15 to .85. The majority of the items selected fell between a range of .4 to .6 in difficulty. All items had a biserial correlation of at least 0.3. Figures 1 and 2 on the following pages show the frequency of items according to difficulty index and biserial correlation respectively. The twelve most discriminating items from Section A, twelve from Section B, and six from Section C, were selected for the final version of the test. Table VIII, on page 51, shows the final distribution of items according to content and relationships. A copy of the pilot study version of the test and a table showing the item discrimination indices and percentage of students passing each item are included in Appendix A.

The items within each category of the final version of the Special Problem-Solving (SPS) Test were arranged in order of ascending difficulty in multiples of five. In each multiple of five questions

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[8] Ibid., page 200.

[9] H.C. Helmstadter, Principles of Psychological Measurement, (New York: Appleton-Century-Crofts 1964), Chapter 7, pp. 157-178.

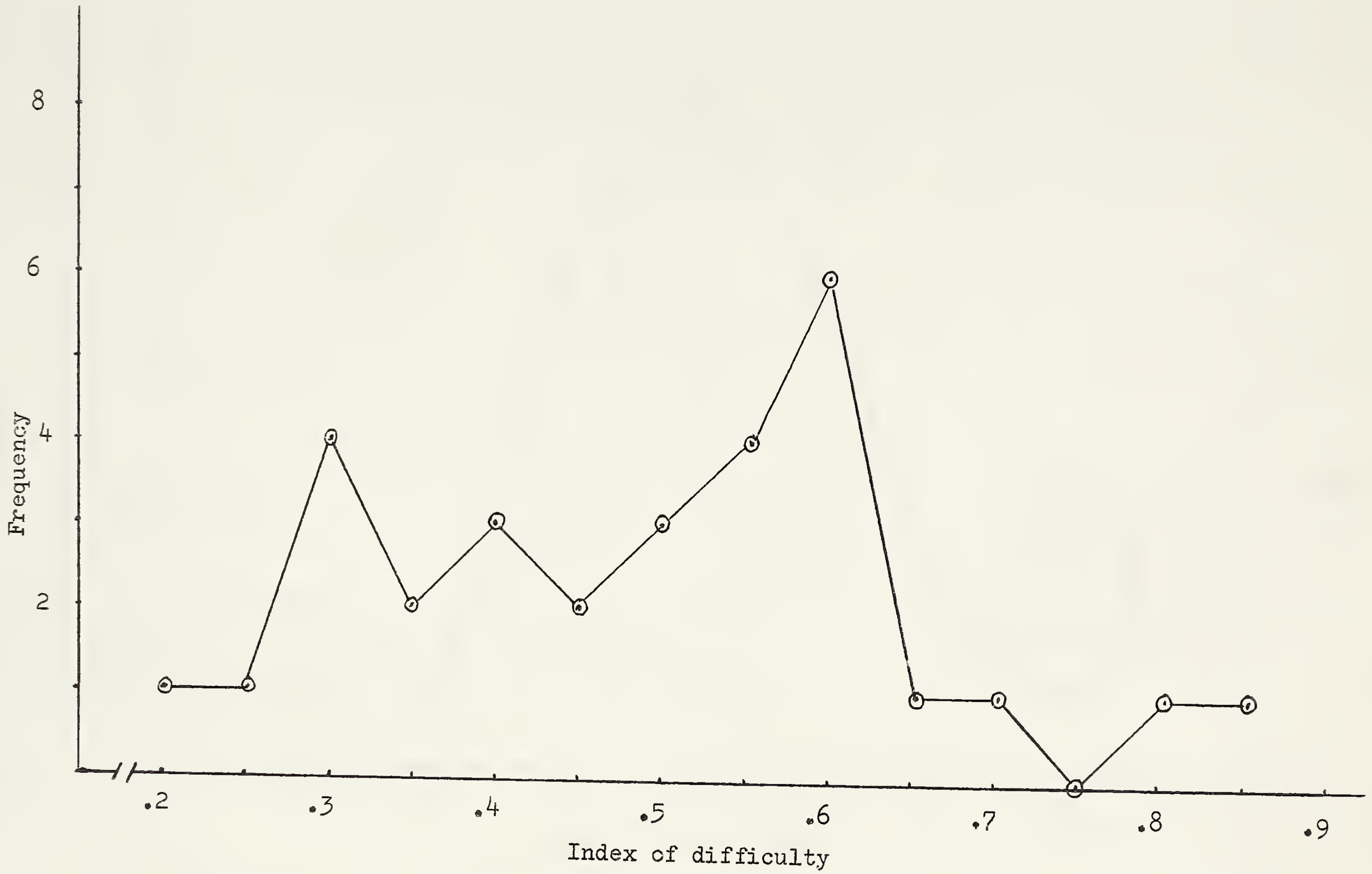

FIGURE 1

ITEM DIFFICULTY ON FINAL VERSION OF SPS TEST
(Range from .204 to .860, Average .489)

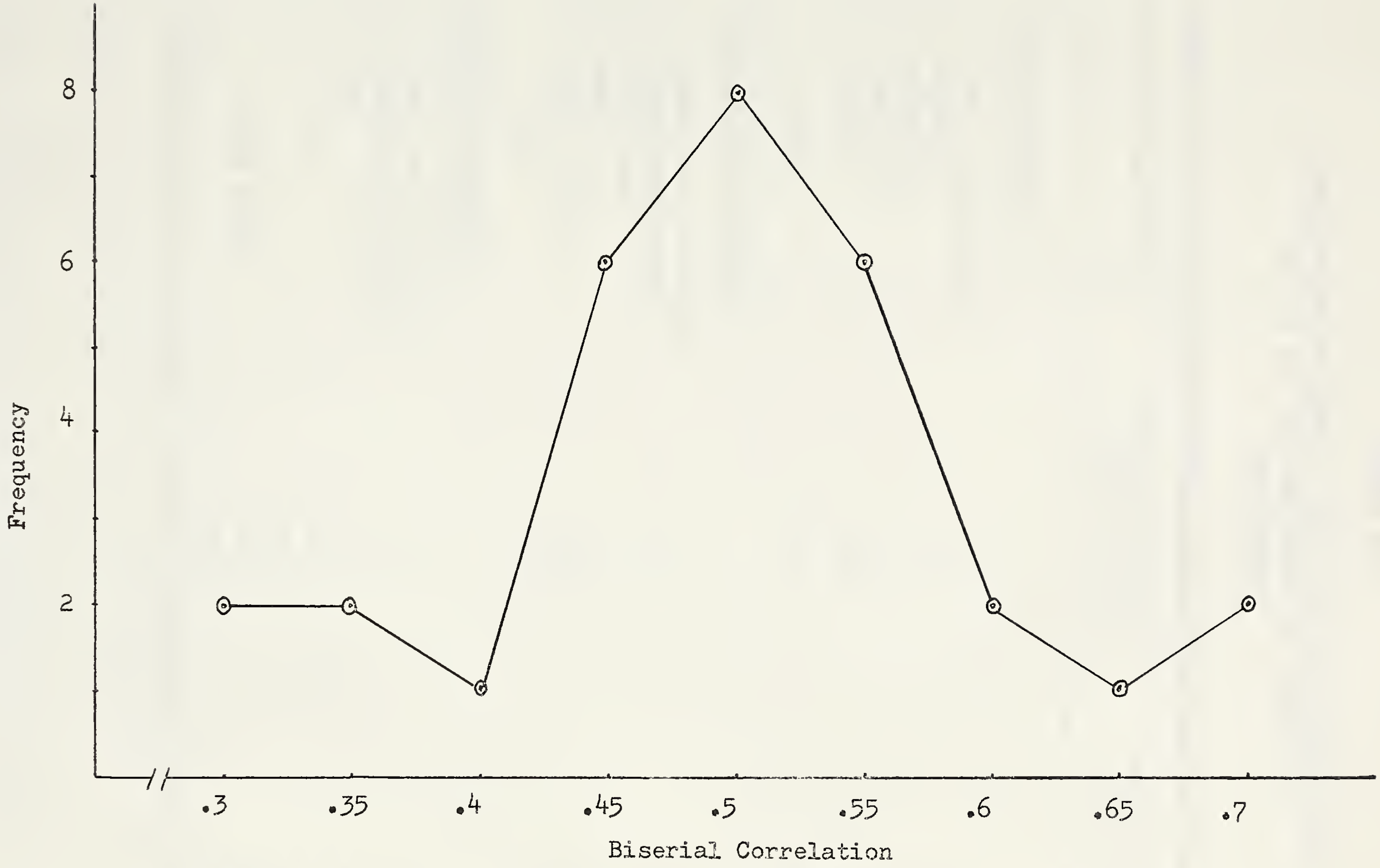

FIGURE 2

ITEM DISCRIMINATION ON FINAL VERSION OF SPS TEST
(Range from .316 to .679, Average .5002)

TABLE VIII

DISTRIBUTION OF ITEMS IN THE PILOT AND FINAL VERSION OF THE SPS TEST ACCORDING TO RELATIONS AND CONTENT

Relations and Content	Number of Items in Pilot Version	Number of Items in Final Version
Linear Relations		
a. Symbol manipulation	8	3
b. Perimeter	4	3
c. Distance	5	2
d. Inequality	4	2
e. Average	5	2
Total	26	12
Proportional Relations		
a. Symbol manipulation	3	2
b. Scale measurement	3	2
c. Percentage	5	3
d. Area	3	2
e. Other context	8	3
Total	22	12
Quadratic Relations		
a. Symbol manipulation	2	1
b. Pythagorean theorem	3	2
c. Area	6	3
Total	11	6
Grand Total	59	30

the first and third questions dealt with linear relations, the second and fourth dealt with proportional relations and every fifth question dealt with quadratic relations. In the event that a student was not able to finish the test in the allotted time no one category would receive a disproportionate number of responses. A copy of the final version of the Special Problem-Solving (SPS) Test is included in Appendix B.

Because the majority of the test items regarding from, structure, and content were derived from Junior High School SMSG and UICSM materials (both of these groups are widely recognized to be influential and authoritative in the field of school mathematics), it is assumed that the Special Problem -Solving (SPS) test has acceptable validity. Furthermore, constructive criticisms of experts in the field of mathematics education were used to validate the test.

The Testing Program

The Cooperative School and College Ability Test (SCAT), Form 4A[10] was administered to all the students involved in the experiment during the 1963-64 school term to provide scores of verbal ability (V) and quantitative ability (Q). The SCAT test is reported in the Fifth Mental Measurements Yearbook[11], as having Kuder-Richardson Formula 20 total reliability score of .95. The test is also described

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[10] Educational Testing Service, Cooperative School and College Ability Tests, School Ability Test, Form 4A, (Princeton: Cooperative Test Division, Educational Testing Service, 1956).

[11] Oscar K. Buros (ed.), The Fifth Mental Measurements Yearbook, (Highland Park, N.J.: The Gryphon Press, 1959), pp. 453-454.

as being non-specific and indicative of a student's ability to achieve at school studies rather than of his actual achievement level.

The grade eight and nine portion of the Arithmetic Problem-Solving (APS) subtest of the IOWA Tests of Basic Skills (IOWA)[12] was also administered to the students involved in the experiment during the 1963-64 school term. The Fifth Mental Measurements Yearbook[13] reports that the total IOWA Arithmetic test has a split-half reliability coefficient of .91 and the problem-solving test has a subtest reliability coefficient of .79. The reviewers state "that computation is encountered in meaningful form in the subtest, although one gets the impression that the range of topics in this test is rather narrow, with over emphasis on money at the expense of items in capacity, weight, and measures".[14] However, the IOWA test manual reports that "in the test on problem solving, competence is tested in a functional setting in problems which have been chosen to be challenging and practical".[15]

In May, 1965, the grade eight and nine portion of the IOWA APS test was readministered to the 492 students included in the experiment. At the same time the SPS test was administered to the students. From the data obtained from the SPS test a Kuder-Richardson Formula 20 reliability index of 0.7990 was calculated. The data used in calculating

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[12] E.F. Lindquist and A.N. Hieronymus, Iowa Tests of Basic Skills, Multi-Level Edition for Grades 3-9, (Boston: Houghton Mifflin Company, 1956), pp. 93-95.

[13] Buros, op. cit., p. 35.

[14] Ibid., page 36.

[15] Iowa Tests of Basic Skills, Manual for Administrators, Supervisors and Councellors, (Boston: Houghton Mifflin Company, 1964), p. 37.

the index of reliability as outline by Ferguson[16] are included in Appendix C. A table showing the item discrimination indices and percentage of the 492 students passing each item of the final version of the SPS test, and the item frequency with respect to discrimination and difficulty is included in Appendix B.

Each student who had written all the tests included in the experiment was assigned an identification number. The September 1963, March 1964, and May 1965, IOWA APS test scores; verbal, quantitative, and total SCAT scores; linear, proportional, quadratic, and total SPS test scores attained by each student together with the students' ID number were punched on IBM cards. The data thus entered on 492 IBM cards were processed as described in the succeeding pages. The raw scores obtained by each student were listed and are included in Appendix C.

Statistical Analysis of Scores Attained During Initial Testing

The total SCAT scores (verbal and quantitative) were used to arbitrarily divide the sample into three scholastic ability levels. This was deemed necessary for assessing the effects of the different textbook programs on students of different scholastic abilities.
A frequency distribution of the 492 total SCAT scores ranging frome 32 to 109 was prepared and the sample was arbitrarily divided into thirds. Consequently, 165 students with scores of 96 or above were classified as scoring in the upper third of scholastic ability. One hundred

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[16] George A. Ferguson, Statistical Analysis in Psychology and Education, (New York: McGraw-Hill Book Co. Inc., 1959), pp. 280-281.

TABLE IX

GROUP AND TOTAL CUMULATIVE PROPORTIONS ON SCAT TOTAL SCORES

SCAT Total Scores	Mid Percentile Rank on SCAT Norms*	CUMULATIVE PROPORTIONS				
		STM	EMM	TRAD	STM-EMM	All Subjects
105-109	99+	1.00	1.00	1.00	1.00	1.00
102-104	97-98	.95	.97	.91	.90	.94
101	96	.86	.91	.86	.75	.86
99-100	94	.82	.90	.84	.71	.83
97-98	92	.76	.86	.78	.61	.77
95-96	90	.69	.78	.70	.56	.70
92-94	86	.61	.67	.63	.46	.61
90-91	81	.52	.56	.50	.34	.50
87-89	76	.44	.46	.41	.29	.42
85-86	72	.40	.38	.35	.27	.37
83-84	68	.35	.32	.33	.25	.33
80-82	65	.30	.26	.32	.19	.29
76-79	58	.21	.17	.22	.19	.20
72-75	52	.11	.09	.12	.17	.12
67-71	45	.05	.04	.10	.10	.07
63-66	38	.03	.03	.06	.08	.05
59-62	32	.02	.03	.02	.04	.02
32-58	6-26	.01	.02	.01	.02	.01

*Cooperative Test division of the Educational Testing Service, Cooperative School and College Ability Tests, Technical Report, (Princeton, N.J.: Educational Testing Service, 1957), p. 40.

sixty-six students with scores from 85 to 95 were classified as scoring in the middle third of scholastic ability and 161 students with scores of 84 or less were classified as scoring in the lower third of scholastic ability. Table IX, on page 55, shows that the students who scored in the upper third of the scholastic ability range fell above the ninetieth percentile on publisher's norms. Since this can be classified as very high scholastic ability, this group is referred to as high ability level (H). Similarly, the middle third is referred to as average (A) and the lower third is referred to as low ability (L). Table X shows the distribution of students in each program according to ability level.

TABLE X

DISTRIBUTION OF STUDENTS BY PROGRAMS AND ABILITY LEVELS

Ability Level	GROUP				
	STM	EMM	TRAD	STM-EMM	Total
H	66	28	45	27	166
A	62	43	48	12	165
L	69	33	46	13	161
Total	197	104	139	52	492

A chi square test[17] carried out on the cell frequencies in Table X indicated that they were not significantly different from being

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[17] George A. Ferguson, Statistical Analysis in Psychology and Education, (New York: McGraw-Hill Book Co. Inc., 1959), pp. 158-178.

proportional to the row and column totals. The calculation of chi square is included in Appendix D.

One way analysis of variance[18] was carried out to test the hypotheses that during the initial testing there are no significant differences among the mean scores obtained by the groups studying the STM, EMM, TRAD, and STM-EMM materials on

(a) the September, 1963, IOWA APS test,
(b) the March, 1964, IOWA APS test,
(c) the September, 1963, SCAT Verbal test,
(d) the September, 1963, SCAT Quantitative test,
(e) the September, 1963, SCAT Total test.

The results of the analysis are summarized in Table XI on page 58.

The critical F ratio at a .01 level of significance associated with the degrees of freedom shown in Table XI is 3.78. This means that there is a significant difference between mean scores at the .01 level if the ratio exceeds 3.78. The use of a .01 level insures that a mean score difference would not be considered significant unless it could only occur as a result of sampling error in less than one out of one hundred such sample mean score comparisons.

Table XI shows that there were no significant differences among the group means scores on the APS-63, APS-64, V, Q, and T tests at the .01 level of significance. A summary of analysis of variance is included in Appendix D.

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[18] _Ibid._, pp. 227-241.

TABLE XI

SUMMARY OF ANALYSIS OF VARIANCE ON SCORES OBTAINED DURING THE INITIAL TESTING

Group		TEST MEANS				
	N	$\overline{X}_{APS-63}$	$\overline{X}_{APS-64}$	$\overline{X}_V$	$\overline{X}_Q$	$\overline{X}_T$
STM	197	14.147	17.487	49.086	40.279	89.401
EMM	104	15.529	18.067	48.904	39.769	88.500
TRAD	139	14.432	18.532	48.504	40.453	88.914
STM-EMM	52	14.885	17.388	50.423	40.981	91.404
Grand Mean				49.024	40.290	89.285
$F = \frac{MS_{Group}}{MS_{Within}}$		1.98	1.26	1.03	0.44	0.79

Critical $F_{.01(3,488)} = 3.78$

$\overline{X}_{APS-63}$ represents a group mean score on the IOWA Arithmetic Problem-Solving Test administered in September, 1963.

$\overline{X}_{APS-64}$ represents a group mean score on the IOWA Arithmetic Problem-Solving Test administered in March, 1964.

$\overline{X}_V$ represents a group mean score on the SCAT Verbal Test.

$\overline{X}_Q$ represents a group mean score on the SCAT Quantitative Test.

$\overline{X}_T$ represents a group mean score on the SCAT Total Test.

Statistical Procedures Used to Analyse Scores Attained During Final Testing

A "t" test outline by Ferguson[19] was used to test whether there areany significant differences between pairs of group means and between pairs of group cell means at the three scholastic ability levels on the SPS, May, 1965, test scores and the IOWA APS, May, 1965, test scores. Since several of the observed differences in the means were discovered to be significant, a two-way unweighted means analysis of variance[20] was regarded suitable to test the following null hypotheses:

1. On a conventional, standardized arithmetic problem-solving test (IOWA) there are no significant differences among:
   (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
   (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.
2. On a Special Mathematics Problem-Solving Test there are no significant differences among:
   (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
   (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.

In the two-way unweighted means analysis of variance the critical F-ratio is for a fixed model as suggested by Winer[21] and Ferguson[22].

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[19] Ibid., pp. 136-138.

[20] B. J. Winer, Statistical Principles in Experimental Design, (New York: McGraw-Hill Book Company, Inc., 1962), pp. 241-244.

[21] Ibid., page 160

[22] Ferguson, op. cit., page 251.

Critical values at the .01 level were used when statistical tests on the differences between means were carried out.

A "t" test as described by Winer[23] was used to determine if there are significant differences between pairs of group means. The explanation of the test statistic is included in Appendix D.

The Newman-Keuls procedure[24] was used to compare ordered cell means within each of the high (H), average (A), and low (L) ability groups. Borderline contrasts as a result of Newman-Keuls procedure were tested by a "t" test[25] to compare the cell means two at a time in order to take the different cell frequencies into consideration. The "t" test formula is included in Appendix D.

The summary and findings of the two-way unweighted means analysis of variance are included in the following chapter of this study.

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[23] Winer, op. cit., page 208.

[24] Winer, op. cit., pp. 238-239.

[25] Winer, op. cit., page 208.

CHAPTER IV

ANALYSIS OF DATA AND RESULTS

The following null hypotheses were tested:

1. On a conventional standardized arithmetic problem-solving test (IOWA) there are no significant differences among:
    - (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
    - (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.
2. On a Special Mathematics Problem-Solving Test there are no significant differences among:
    - (a) the group mean scores attained by EMM, STM, STM-EMM, and TRAD students.
    - (b) the scholastic ability level (SCAT) mean scores of the EMM, STM, STM-EMM, and TRAD students.

Part I of this chapter contains the results and interpretations of the statistical tests when hypothesis 1 is tested, while Part II contains the results and interpretations of the statistical tests when hypothesis 2 is tested. Since the Special Problem-Solving Test was designed to test problem-solving proficiency in three algebraic relations categories, i.e. linear, proportional, and quadratic, a two-way unweighted means analysis of variance on the scores obtained by students in respective categories is also summarized in Part II.

The null hypotheses tested are stated before the results of the statistical tests and interpretations are presented.

PART I

Null Hypothesis 1

On the IOWA APS test administered in May, 1965, there are no significant differences

(a) among the group mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(b) among the scholastic ability level mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(c) attributed to interaction.

The IOWA APS test cell and group means are presented in Table XII. The analysis of variance carried out on these scores appears in Table XIII.

Since the F-ratio (3.384) when making comparisons among group mean IOWA APS scores did not exceed the critical value (3.78), null hypothesis 1 (a) was accepted. The differences among the group IOWA APS test mean scores were not significant.

Since the F-ratio (114.619) for comparisons among ability level IOWA APS test mean scores was larger than the critical value (4.61), null hypothesis 1 (b) was rejected. Significant differences among the ability level IOWA APS test mean scores were observed.

Since the F-ratio (1.617) attributed to interaction did not exceed the critical value (2.80), null hypothesis 1 (c) was accepted. The interaction affect was not statistically significant.

The group means were not compared two at a time, since there were no significant differences among the four group mean scores on the IOWA APS test.

TABLE XII

CELL AND GROUP MEANS OF THE IOWA APS TEST SCORES

Ability Level	GROUP			
	EMM	STM	STM-EMM	TRAD
H	25.25	26.12	24.07	25.60
A	20.21	21.82	20.50	21.67
L	18.70	17.01	14.62	16.15
Group Mean	21.09	21.58	20.89	21.12

Grand Mean = 21.27

TABLE XIII

SUMMARY OF ANALYSIS OF VARIANCE ON IOWA APS TEST SCORES

SOURCE	SS	MS	DF	F
Ability	4513.89	2256.94	2	114.619
Group	199.91	66.64	3	3.384
Interaction	191.60	31.93	6	1.617
Within	9451.73	19.69	480	

Critical F-ratios

$F_{.01}$ (2,480) = 4.61, $F_{.01}$ (3,480) = 3.78, $F_{.01}$ (6,480) = 2.80.

PART II

Null Hypothesis 2

On the SPS test administered in May, 1965, there are no significant differences

(a) among the group mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(b) among the scholastic ability level mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(c) attributed to interaction.

The SPS test cell and group means are presented in Table XIV. The analysis of variance carried out on these scores appears in Table XV.

Since the F-ratio (4.689) when making comparisons among group mean SPS test scores was larger than the critical value (3.78), null hypothesis 2 (a) was rejected. Significant differences among the group SPS test mean scores were observed.

Since the F-ratio (139.917) for comparisons among ability level SPS test mean scores was larger than the critical value (4.61), null hypothesis 2 (b) was rejected. The differences among the ability level SPS test mean scores were significant.

Since the observed F-ratio (2.307) attributed to interaction did not exceed the critical value (2.80), null hypothesis 2 (c) was accepted. The interaction factor was not significant.

Since there were significant differences among the group mean scores on the SPS test, it was necessary to compare group means two at a time by the use of a "t" test.

TABLE XIV

CELL AND GROUP MEANS OF THE SPS TEST SCORES

Ability Level	GROUP			
	EMM	STM	STM-EMM	TRAD
H	19.54	18.68	20.52	19.71
A	15.34	14.10	15.25	14.46
L	14.15	9.91	9.15	9.09
Group Means	16.10	14.17	16.46	14.38

Grand Mean = 14.88

TABLE XV

SUMMARY OF ANALYSIS OF VARIANCE ON SPS TEST SCORES

SOURCE	SS	MS	DF	F
Ability	4941.23	2470.61	2	139.917
Group	248.40	82.80	3	4 .689
Interaction	244.40	40.73	6	2.307
Within	8475.69	17.66	480	

Critical F-ratios

$F_{.01}(2,480) = 4.61$, $F_{.01}(3,480) = 3.78$, $F_{.01}(6,480) = 2.80$.

Comparisons of Group Mean Scores on the SPS Test, Null Hypothesis 2(a)

On the SPS test, there are no significant differences between the group mean scores obtained by: (i) EMM and STM, (ii) EMM and STM-EMM, (iii) EMM and TRAD, (iv) STM and STM-EMM, (v) STM and TRAD, (vi) STM-EMM and TRAD students.

The results of the "t" test explained by Winer[1] appear in Table XVI below.

TABLE XVI

COMPARISONS OF GROUP MEAN SCORES ON THE SPS TEST

Group	EMM	STM	STM-EMM	TRAD
EMM		3.58*	.67	3.19*
STM			4.24*	.39
STM-EMM				3.87*

Critical value $t_{.01}(480) = 2.33$.

* significant at the .01 level.

Parts (i), (iii), (iv), and (vi) of hypothesis 2 (a) were rejected, and parts (ii) and (v) of the hypothesis 2 (a) were accepted.

The group mean scores on the SPS test obtained by the EMM and STM-EMM groups were significantly higher than the group mean scores obtained by the STM and TRAD groups. The group mean score obtained

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[1] B. J. Winer, Statistical Principles in Experimental Design, (New York: McGraw-Hill Book Company, Inc., 1962), pp. 208-209.

by the EMM group on the SPS test was not significantly different from the group mean score obtained by the STM-EMM group. Furthermore, the group mean score obtained by the STM group on the SPS test was not significantly different from the group mean score obtained by the TRAD group.

Analysis of Cell Mean Scores on the SPS Test

The Newman-Keuls procedure[2] was used to make comparisons among group cell means at each ability level. In Table XVII on page 68, the cell means of the four groups are arranged in ascending order from left to right and from top to bottom. The absolute differences and critical differences, in parentheses, appear in the respective cells. If the critical difference is exceeded by the absolute difference, then the means are considered to be significantly different.

The critical differences between means is determined by the following formula: $\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$, where $MS_{w.cell}$ is the pooled within-cell variance, n_h is the harmonic mean of the cell frequencies, and q is the studentized range statistic. The studentized range statistic is the function of the number of steps between the means when they are arranged on an ordered scale. In Table XVII the steps are represented by r. The critical level is .01 and 480 is the number of degrees of freedom.

Table XVII shows that there are no significant differences between pairs of cell group means at the high ability level.

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[2] *Ibid.*, pp. 238-239.

The findings of the Newman-Keuls procedure as applied to the average ability cell means are shown in XVIII on page 69. It is also found that the differences between pairs of the average ability cell group means are not significantly different.

Table XIX on page 70, summarizes the findings of the Newman-Keuls procedure as applied to the low ability cell group means. It is found that the EMM low ability cell mean on the SPS test is significantly higher than that of the TRAD, STM-EMM, and STM cell means. There are no significant differences among the cell means of the TRAD, STM, and STM-EMM at the low ability level.

TABLE XVII

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE HIGH ABILITY GROUP ON THE SPS TEST

GROUP		STM	EMM	TRAD	STM-EMM
	MEANS	18.68	19.54	19.71	20.52
STM	18.68	- -	.86 (2.76)	1.03 (3.13)	1.84 (3.34)
EMM	19.54		- -	.17 (2.76)	.98 (3.13)
TRAD	19.71			- -	.81 (2.76)

	r = 2	r = 3	r = 4
$q_{.01}(r, 480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	2.76	3.13	3.34

TABLE XVIII

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE AVERAGE ABILITY GROUP ON THE SPS TEST

GROUP		STM	TRAD	STM-EMM	EMM
	MEANS	14.10	14.46	15.25	15.34
STM	14.10	- -	.36 (2.76)	1.15 (3.13)	1.24 (3.34)
TRAD	14.46		- -	.79 (2.76)	.88 (3.13)
STM-EMM	15.25			- -	.09 (2.76)

	r = 2	r = 3	r = 4
$q_{.01}(r, 480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	2.76	3.13	3.34

TABLE XIX

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE LOW ABILITY GROUP ON THE SPS TEST

GROUP		TRAD	STM-EMM	STM	EMM
	MEANS	9.09	9.15	9.91	14.15
TRAD	9.09	- -	.06 (2.76)	.82 (3.13)	5.06* (3.34)
STM-EMM	9.15		- -	.76 (2.76)	5.00* (3.13)
STM	9.91			- -	4.23* (2.76)

	r = 2	r = 3	r = 4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right]q$	2.76	3.13	3.34

The foregoing analysis of cell mean scores reveals that there were no significant differences among the scores obtained by the four groups at the high and average ability levels, but at the low ability level the EMM students scored significantly higher on the SPS test than the TRAD, STM, and STM-EMM students. A graphic illustration of the relationships among the SPS test cell mean scores for the four groups appears in Figure 3 on page 72.

Null Hypothesis 2

On linear relations section of the SPS test there are no significant differences:

(a) among the group mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(b) among the scholastic ability level mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(c) attributed to interaction.

The SPS test (linear section) cell and group means are presented in Table XX. The analysis of variance carried out on these scores appears in Table XXI.

On examination of the F-ratio (3.415) when making comparisons among group mean SPS (linear section) scores it was observed that this ratio was close to the critical F-ratio (3.78). Furthermore,, on examining the one-way analysis of variance (included in Appendix D) the comparison among the group mean scores resulted in a F-ratio of (5.22) which was significant at the .01 level. These observations prompted the investigator to reject null hypothesis 2 (a) and to proceed with the comparison of group mean scores and ability cell mean scores.

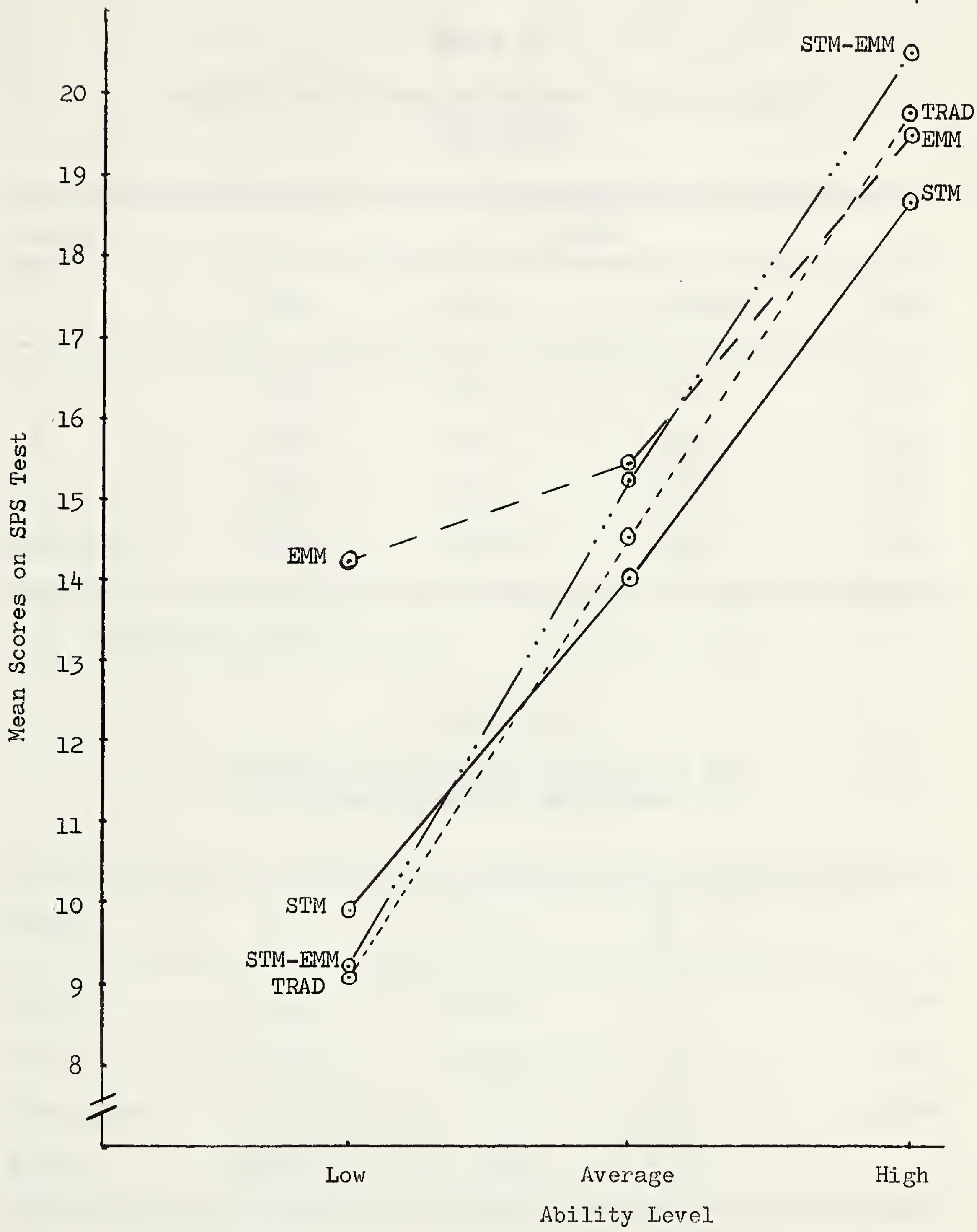

FIGURE 3

CELL MEAN SPS SCORES FOR THE FOUR GROUPS AT THE THREE ABILITY LEVELS

TABLE XX

CELL AND GROUP MEANS OF THE SPS (LINEAR SECTION) TEST SCORES

Ability Level	GROUP			
	EMM	STM	STM-EMM	TRAD
H	7.25	6.91	8.11	7.31
A	5.44	4.89	5.67	5.17
L	5.09	3.51	3.62	3.59
Group Mean	5.82	5.08	6.42	5.34

Grand Mean = 5.45

TABLE XXI

SUMMARY OF ANALYSIS OF VARIANCE ON SPS (LINEAR SECTION) TEST SCORES

SOURCE	SS	MS	DF	F
Ability	729.07	364.54	2	92.907
Group	40.19	13.40	3	3.415
Interaction	46.25	7.71	6	1.964
Within	1883.37	3.92	480	

Critical F-ratios

$F_{.01}(2,480) = 4.61$, $F_{.01}(3,480) = 3.78$, $F_{.01}(6,480) = 2.80$.

Since the F-ratio (92.907) for comparisons among ability level SPS (linear section) test means scores was larger than the critical value (4.61), null hypothesis 2 (a) was rejected. Significant differences among the ability level mean scores were observed.

Since the observed F-ratio (1.964) attributed to interaction did not exceed the critical value (2.80), null hypothesis 2 (c) was accepted. The interaction factor was not significant.

Since there were significant differences among the group mean scores on the SPS (linear section) test on the basis of the one-way analysis, the group means were compared two at a time by the use of a "t" test.

Comparisons of Group Mean Scores on the SPS (linear section) Test,

On the SPS (linear section) test, there are no significant differences between the group mean scores obtained by: (i) EMM and STM, (ii) EMM and STM-EMM, (iii) EMM and TRAD, (iv) STM and STM-EMM, (v) STM and TRAD, (vi) STM-EMM and TRAD students.

The results of the "t" test appear in Table XXII. Parts (i), (iv), and (vi) of hypothesis 2 (a) were rejected, and parts (ii), (iii), and (v) of hypothesis 2 (a) were accepted.

The group mean score on the SPS (linear section) test obtained by the EMM group was significantly higher than the group mean score obtained by the STM group. The group mean score on the SPS (linear section) test obtained by the STM-EMM group was significantly higher than the group mean scores obtained by the STM and TRAD groups. The group mean scores obtained by the EMM and STM-EMM groups were not

significantly different. The group mean scores obtained by the EMM and TRAD groups were not significantly different. Furthermore, the group mean scores obtained by the STM and TRAD groups were not significantly different.

TABLE XXII

COMPARISONS OF GROUP MEAN SCORES ON THE SPS (LINEAR SECTION) TEST

GROUP	EMM	STM	STM-EMM	TRAD
EMM		2.85*	2.31	1.85
STM			5.15*	1.00
STM-EMM				4.15*

Critical value $t_{.01}(480) = 2.33$.

* significant at the .01 level.

Analysis of Cell Mean Scores on SPS (Linear Section) Test

The Newman-Keuls procedure was used to make comparisons among group cell means at each ability level. Table XXIII on page 77 shows that there are no significant differences between pairs of cell group means at the high ability. However, by considering the effects of using the two actual cell frequencies rather than n_h in the calculation of the critical mean difference, it was discovered that the contrast between the high ability mean scores on the SPS (linear section) for the STM and STM-EMM groups was borderline. The "t" test gave the

following results: STM vs. STM-EMM $t=2.79$ and $t_{.01}(480) = 2.33$. The investigator concluded that the difference between mean SPS (linear section) scores obtained by the STM and STM-EMM high ability level students was significant and that the STM-EMM cell mean was higher.

The findings of the Newman-Keuls procedure as applied to the average ability cell means are shown in Table XXIV on page 78. The differences between pairs of the average ability cell group means were not significantly different.

Table XXV, on page 79, summarizes the findings of the Newman-Keuls procedure as applied to the low ability cell group means. It was found that the EMM low ability cell mean on the SPS (linear section) test was significantly higher than the cell means of the STM, STM-EMM, and TRAD groups. The differences among the cell mean scores obtained by the STM, STM-EMM, and TRAD groups were not significantly different.

The foregoing analysis of cell mean scores reveals that there was a significant difference between the scores obtained by the STM-EMM and STM groups at the high ability level in favor of the STM-EMM group; that there were no significant differences among the scores obtained by the four groups at the average ability level; and that the EMM students scored significantly higher on the SPS (linear section) test than the STM, TRAD, and STM-EMM students at the low ability level. A graphic illustration of the relationships among the SPS (linear section) test cell mean scores for the four groups appears in Figure 4 on page 80.

TABLE XXIII

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE HIGH ABILITY GROUP ON THE SPS (LINEAR SECTION) TEST

GROUP		STM	EMM	TRAD	STM-EMM
	MEANS	6.91	7.25	7.31	8.11
STM	6.91	- -	.34 (1.28)	.40 (1.44)	1.20 (1.54)
EMM	7.25		- -	.06 (1.28)	.86 (1.44)
TRAD	7.31			- -	.80 (1.28)

	r = 2	r = 3	r = 4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	1.28	1.44	1.54

TABLE XXIV

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE AVERAGE ABILITY GROUP ON THE SPS (LINEAR SECTION) TEST

GROUP		STM	TRAD	EMM	STM-EMM
	MEANS	4.89	5.17	5.44	5.67
STM	4.89	- -	.28 (1.28)	.55 (1.44)	.78 (1.54)
TRAD	5.17		- -	.27 (1.28)	.50 (1.44)
EMM	5.44			- -	.23 (1.28)

	r = 2	r = 3	r = 4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	1.28	1.44	1.54

TABLE XXV

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE LOW ABILITY GROUP ON THE SPS (LINEAR SECTION) TEST

GROUP		STM	TRAD	STM-EMM	EMM
	MEANS	3.51	3.59	3.62	5.09
STM	3.51	- -	.08 (1.28)	.11 (1.44)	1.58* (1.54)
TRAD	3.59		- -	.03 (1.28)	1.50* (1.44)
STM-EMM	3.62			- -	1.47* (1.28)

	r = 2	r = 3	r = 4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	1.28	1.44	1.54

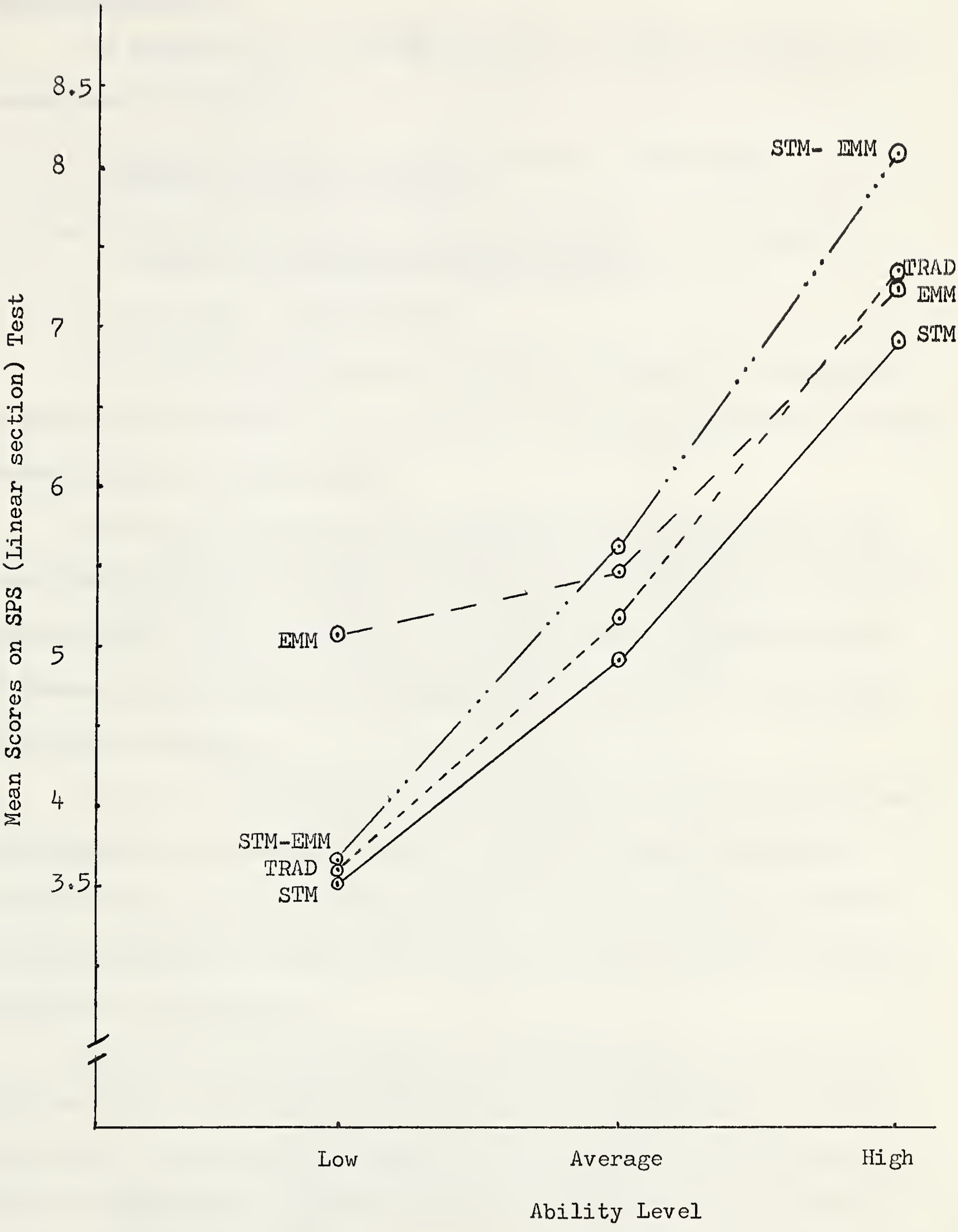

FIGURE 4

CELL MEAN SPS (LINEAR SECTION) SCORES FOR THE FOUR GROUPS AT THE THREE ABILITY LEVELS

Null Hypothesis 2

On proportional relations section of the SPS test there are no significant differences:

(a) among the group mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(b) among the scholastic ability level mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(c) attributed to interaction.

The SPS (proportional section) test cell and group means are presented in Table XXVI. The analysis of variance carried out on these scores appears in Table XXVII.

Since the F-ratio (1.541) when making comparisons among group mean SPS (proportional section) scores was not larger than the critical value (2.78), null hupothesis 2 (a) was accepted. Significant differences among the group SPS (proportional section) test mean scores were not observed.

Since the F-ratio (125.778) for comparisons among ability level SPS (proportional section) test mean scores was larger than the critical value (4.61), null hypothesis 2 (b) was rejected. The differences among the ability level SPS (proportional section) test mean scores were significant.

Since the observed F-ratio (2.275) attributed to interaction did not exceed the critical value (2.80), null hypothesis 2 (c) was accepted. The interaction factor was not significant. Since there were no significant differences among the group mean scores on the SPS (proportional section) test, it was not necessary to compare group means two at a time by the use of a "t" test.

TABLE XXVI

CELL AND GROUP MEANS OF THE SPS (PROPORTIONAL SECTION) TEST SCORES

Ability Level	GROUP			
	EMM	STM	STM-EMM	TRAD
H	8.71	8.61	9.07	9.08
A	7.33	7.03	7.25	7.13
L	6.09	5.00	4.38	4.33
Group Mean	7.31	6.85	7.49	6.84

Grand Mean = 7.01

TABLE XXVII

SUMMARY OF ANALYSIS OF VARIANCE ON SPS (PROPORTIONAL SECTION) TEST SCORES

SOURCE	SS	MS	DF	F
Ability	934.83	467.41	2	125.778
Group	17.18	5.73	3	1.541
Interaction	50.73	8.45	6	2.275
Within	1783.38	3.72	480	

Critical F-ratios

$F_{.01}(2,480) = 4.61$, $F_{.01}(3,480) = 3.78$, $F_{.01}(6,480) = 2.80$.

Null Hypothesis 2

On quadratic relations section of the SPS test there are no significant differences:

(a) among the group mean scores obtained by EMM, STM, STM-EMM, and TRAD students.

(b) among the scholastic ability level mean scores obtained by EMM, STM, STM-EMM and TRAD students.

(c) attributed to interaction.

The SPS (quadratic section) test cell and group means are presented in Table XXVIII. The analysis of variance carried out on these scores appears in Table XXIX.

Since the F-ratio (7.538) when making comparisons among group mean SPS (quadratic section) test scores was larger than the critical value (3.78), null hypothesis 2 (a) was rejected. Significant differences among the group SPS (quadratic section) mean scores were observed.

Since the F-ratio (44.666) for comparisons among ability level SPS (quadratic section) test mean scores was larger than the critical value (4.61), null hypothesis 2 (b) was rejected. The differences among the ability level SPS (quadratic section) test mean scores were significant. Since the observed F-ratio (2.658) attributed to interaction did not exceed the critical value (2.80), null hypothesis 2 (c) was accepted. The interaction factor was not significant.

Since there were significant differences among the group mean scores on the SPS (quadratic section) test, it was necessary to compare group means two at a time by the use of a "t" test.

TABLE XXVIII

CELL AND GROUP MEANS OF THE SPS (QUADRATIC SECTION) TEST SCORES

Ability Level	GROUP			
	EMM	STM	STM-EMM	TRAD
H	3.57	3.17	3.33	3.31
A	2.58	2.16	2.33	2.15
L	2.97	1.29	1.15	1.26
Group Mean	2.97	2.19	2.56	2.23

Grand Mean = 2.41

TABLE XXIX

SUMMARY OF ANALYSIS OF VARIANCE ON SPS (QUADRATIC SECTION) TEST SCORES

SOURCE	SS	MS	DF	F
Ability	173.25	86.62	2	44.666
Group	43.86	14.62	3	7.538
Interaction	30.94	5.16	6	2.658
Within	930.90	1.94	480	

Critical F-ratios

$F_{.01}(2,480) = 4.61$, $F_{.01}(3,480) = 3.78$, $F_{.01}(6,480) = 2.80$.

Comparisons of Group Mean Scores on the SPS (Quadratic section) Test, Null Hypothesis 2 (a)

On the SPS (quadratic section) test, there are no significant differences between the group mean scores obtained by: (i) EMM and STM, (ii) EMM and STM-EMM, (iii) EMM and TRAD, (iv) STM and STM-EMM, (v) STM and TRAD, (vi) STM-EMM and TRAD students.

The results of the "t" test appear in Table XXX below.

TABLE XXX

COMPARISONS OF GROUP MEANS SCORES ON THE SPS (QUADRATIC SECTION) TEST

GROUP	EMM	STM	STM-EMM	TRAD
EMM		4.33*	2.28	4.11*
STM			2.06	0.22
STM-EMM				1.83

Critical value $t_{.01}(480) = 2.33$.

* significant at the .01 level.

Parts (i) and (iii) of hypothesis 2 (a) were rejected, and parts (ii), (iv), (v), and (vi) of hypothesis 2 (a) were accepted.

The group mean score obtained by the EMM students on the SPS (quadratic section) test was significantly higher from the group mean scores obtained by the STM and TRAD groups. There was no significant difference between the EMM and STM-EMM group mean scores. Furthermore, there were no significant differences among the mean scores obtained

by the STM, STM-EMM, and TRAD groups on the SPS (quadratic section) test.

Analysis of Cell Mean Scores on SPS (Quadratic section) Test

The Newman-Keuls procedure was used to make comparisons among group cell means at each ability level. Tables XXXI and XXXII on pages 87 and 88 respectively, show that there are no significant differences between pairs of cell group means at the high and average ability levels.

The findings of the Newman-Keuls procedure as applied to the low ability cell means are shown in Table XXXIII on page 89. It was found that the EMM low ability cell mean on the SPS (quadratic section) test was significantly higher than the cell means of the STM, STM-EMM, and TRAD groups. The differences among the cell mean scores obtained by the STM, STM-EMM, and TRAD groups were not significantly different.

The foregoing analysis of cell mean scores reveals that there were no significant differences among the scores obtained by the four groups at the high and average ability levels. However, the EMM students scored significantly higher on the SPS (quadratic section) test than the STM, STM-EMM, and TRAD students at the low ability level. A graphic illustration of the relationships among the SPS (quadratic section) test cell mean scores for the four groups appears in Figure 5 on page 90.

TABLE XXXI

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE HIGH ABILITY GROUP ON THE SPS (QUADRATIC SECTION) TEST

GROUP		STM	TRAD	STM-EMM	EMM
	MEANS	3.17	3.31	3.33	3.57
STM	3.17	- -	.14 (.91)	.16 (1.03)	.40 (1.10)
TRAD	3.31		- -	.02 (.91)	.26 (1.03)
STM-EMM	3.33				.24 (.91)

	r = 2	r = 3	r = 4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right] q$	.91	1.03	1.10

TABLE XXXII

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE AVERAGE ABILITY GROUP ON THE SPS (QUADRATIC SECTION) TEST

GROUP		TRAD	STM	STM-EMM	EMM
	MEANS	2.15	2.16	2.33	2.58
TRAD	2.15	- -	.01 (.91)	.18 (1.03)	.43 (1.01)
STM	2.16		- -	.17 (.91)	.42 (1.03)
STM-EMM	2.33				.25 (.91)

	r=2	r=3	r=4
$q_{.01}(r,480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right]q$	.91	1.03	1.10

TABLE XXXIII

ABSOLUTE AND CRITICAL DIFFERENCES BETWEEN ORDERED CELL MEANS OF THE LOW ABILITY GROUP ON THE SPS (QUADRATIC SECTION) TEST

GROUP		STM-EMM	TRAD	STM	EMM
	MEANS	1.15	1.26	1.29	2.97
STM-EMM	1.15	- -	.11 (.91)	.14 (1.03)	1.82* (1.10)
TRAD	1.26		- -	.03 (.91)	1.71 * (1.03)
STM	1.29			- -	1.68 * (.91)

	r = 2	r = 3	r = 4
$q_{.01}(r, 480)$	3.64	4.12	4.40
$\left[\sqrt{(MS_{w.cell}/n_h)}\right]q$	.91	1.03	1.10

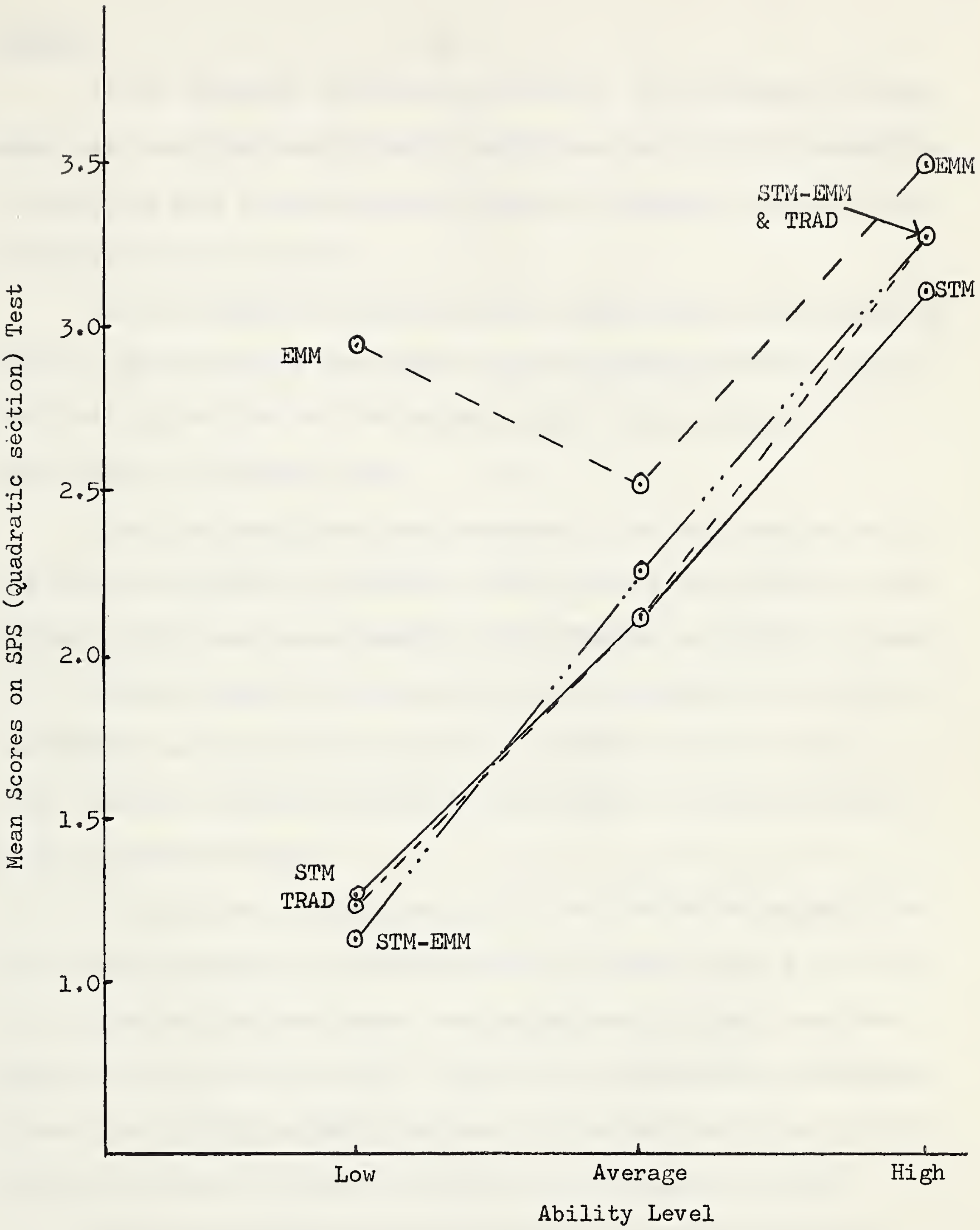

FIGURE 5

CELL MEAN SPS (QUADRATIC SECTION) SCORES FOR THE FOUR GROUPS AT THE THREE ABILITY LEVELS

[illegible]

Summary

In the treatment of data in this study, the difference between means was considered statistically significant only if the probability of observing such a difference as a result of sampling error was one or less out of one hundred.

In the comparisons between pairs of group means on the IOWA APS test the STM mean score was higher than the TRAD mean score which, in turn, was higher than the STM-EMM mean score which, in turn, was higher than the EMM mean score.

A two-way unweighted means analysis of variance carried out on the scores obtained by the groups on the IOWA APS test showed no significant differences among the EMM, STM, STM-EMM, and TRAD group means.

In the comparisons between pairs of group means on the SPS test, the STM-EMM mean score was higher than the EMM mean score which, in turn, was higher than the TRAD mean score which, in turn, was higher than the STM mean score.

The differences among group means were tested for significance by a two-way unweighted means analysis of variance and by a "t" test. The STM-EMM and EMM group means were significantly higher than the TRAD and STM group mean scores. There was no significant difference between the STM-EMM and EMM group mean scores; and there was no significant difference between the TRAD and STM group mean scores.

In the comparisons between pairs of cell means at the high ability level on the SPS test, the STM-EMM cell mean was higher than the TRAD cell mean which, in turn, was higher than the EMM cell mean

which, in turn, was higher than the STM cell mean.

The differences among the cell means at the high ability level were tested for significance by following the Newman-Keuls procedure. It was found that there were no significant differences among the group cell means at the high ability level on the SPS test.

In the comparisons between pairs of cell means at the average ability level on the SPS test, the EMM cell mean was higher than the STM-EMMcell mean which, in turn, was higher than the TRAD cell mean which, in turn, was higher than the STM cell mean.

The differences among the cell means at the average ability level were tested for significance by following the Newman-Keuls procedure. It was found that there were no significant differences among the group cell means at the average ability level on the SPS test.

In the comparisons between pairs of cell means at the low ability level on the SPS test, the EMM cell mean was higher than the STM cell mean which, in turn, was higher than the STM-EMM cell mean which, in turn, was higher than the TRAD cell mean.

The differences among the cell means at the low ability level were tested for significance by following the Newman-Keuls procedure. It was found that the EMM cell mean at the low ability level on the SPS test was significantly higher than the STM, STM-EMM, and TRAD cell means.

In the comparisons between pairs of group means on the SPS (linear section) test, the STM-EMM mean score was higher than the

EMM mean score which, in turn, was higher than the TRAD mean score which, in turn, was higher than the STM mean score.

The differences among group means were tested for significance by a two-way unweighted means analysis of variance and by a "t" test. It was found that the EMM group mean score on the SPS (linear section) test was significantly higher than the STM group mean score, and that the STM-EMM group mean score was significantly higher than the STM and TRAD group mean scores. The remaining group mean score differences were not significant.

In the comparisons between pairs of cell means at the high ability level on the SPS (linear section) test, the STM-EMM cell mean was higher than the TRAD cell mean which, in turn, was higher than the EMM cell mean which, in turn, was higher than the STM cell mean.

The differences among the cell means at the high ability level were tested for significance by following the Newman-Keuls procedure. It was found that the STM-EMM cell mean on the SPS (linear section) test was significantly higher than the STM cell mean. The remaining group cell mean differences were not significant.

In the comparisons between pairs of cell means at the average ability level on the SPS (linear section) test, the STM-EMM cell mean was higher than the EMM cell mean which, in turn, was higher than the TRAD cell mean which, in turn, was higher than the STM cell mean.

The differences among the cell means at the average ability level were tested for significance by following the Newman-Keuls procedure. It was found that the group cell means at the average

ability level on the SPS (linear section) test were not significantly different.

In the comparisons between pairs of cell means at the low ability level on the SPS (linear section) test, the EMM cell mean was higher than the STM-EMM cell mean which, in turn, was higher than the TRAD cell mean which, in turn, was higher than the STM cell mean.

The differences among the cell means at the low ability level were tested for significance. It was found that the EMM cell mean at the low ability level on the SPS (linear section) test was significantly higher than the STM, STM-EMM, and TRAD cell means. The remaining group cell mean differences were not significant.

In the comparisons between pairs of group means on the SPS (proportional section) test, the TRAD mean score was higher than the STM-EMM mean score which, in turn, was higher than the EMM mean score which, in turn, was higher than the STM mean score.

A two-way unweighted means analysis of variance carried out on the scores obtained by the groups on the SPS (proportional section) test showedno significant differences among the group means.

In the comparisons between pairs of group means on the SPS (quadratic section) test, the EMM mean score was higher than the STM-EMM mean score which, in turn, was higher than the TRAD mean score which, in turn, was higher than the STM mean score.

The differences among group means were tested for significance by a two-way unweighted means analysis of variance and by a "t" test. The EMM group mean was significantly higher than the STM and TRAD group

means. The remaining group mean differences were not significant.

In the comparisons between pairs of cell means at the high ability level on the SPS (quadratic section) test, the EMM cell mean was higher than the STM-EMM cell mean which, in turn, was higher than the TRAD cell mean which, in turn, was higher than the STM cell mean.

The differences among the cell means at the high ability level were tested for significance and it was found that the group cell means were not significantly different.

In the comparisons between pairs of cell means at the average ability level on the SPS (quadratic section) test, the EMM cell mean was higher than the STM-EMM cell mean which, in turn, was higher than the STM cell mean which, in turn, was higher than the TRAD cell mean.

The differences among the cell means at the average ability level were tested for significance and it was found that the group cell means were not significantly different.

In the comparisons between pairs of cell means at the low ability level on the SPS (quadratic section) test, the EMM cell mean was higher than the STM cell mean which, in turn, was higher than the TRAD cell mean which, in turn, was higher than the STM-EMM cell mean.

The differences among the cell means at the low ability level were tested for significance and it was found that the EMM cell mean was significantly higher than the STM, TRAD, and STM-EMM cell means. The remaining group cell mean differences were not significant.

CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

While many factors may affect mathematical problem-solving proficiency, this study considered only the factors of textbook materials, scholastic ability, initial problem-solving proficiency, time devoted to the study of mathematics, and professional training and experience of teachers.

One-way analysis of variance was used to show that the students' scholastic ability and initial problem-solving proficiency were not significantly different. Data provided by teachers' questionnaire revealed that there were no appreciable differences in the time devoted to the study of mathematics and in the amount of teachers' professional training and experience. The acceptability of each of the hypotheses associated with the May, 1965, IOWA Arithmetic Problem-Solving and the Special Problem-Solving test scores were tested by carrying out a two-way analysis of variance, by appropriate "t" tests, and by using the Newman-Keuls procedures on scores obtained by the students on these tests.

CONCLUSIONS

The conclusions which apply to the nineteen classes of 492 Alberta Grade nine students are as follows:

1. On a conventional arithmetic problem-solving test at the end of grade nine, students studying Seeing Through Mathematics and/or

Exploring Modern Mathematics textbooks in Junior High School achieved equally well as students studying conventional currently authorized textbook materials in Junior High School.

2. On a conventional arithmetic problem-solving test at the end of grade nine, students studying Exploring Modern Mathematics textbooks in Junior High School achieved equally well as students studying Seeing Through Mathematics textbooks in Junior High School.
3. On a conventional arithmetic problem-solving test at the end of grade nine, students studying Seeing Through Mathematics textbooks till the end of grade eight and Exploring Modern Mathematics textbooks in grade nine achieved equally well as students studying only Seeing Through Mathematics textbooks or only Exploring Modern Mathematics textbooks in Junior High School.
4. On a special problem-solving test at the end of grade nine, students studying Seeing Through Mathematics textbooks till the end of grade eight and Exploring Modern Mathematics textbooks in grade nine achieved equally well as students studying only Exploring Modern Mathematics textbooks in Junior High School.
5. On a special problem-solving test at the end of grade nine, students studying Seeing Through Mathematics textbooks till the end of grade eight and Exploring Modern Mathematics textbooks in grade nine achieved significantly higher scores than the students stuyding only Seeing Through Mathematics textbooks or the

students studying currently authorized textbooks in Junior High School.

6. On a special problem-solving test at the end of grade nine, students studying Exploring Modern Mathematics textbooks in Junior High School achieved significantly higher scores than the students studying Seeing Through Mathematics textbooks or the students studying currently authorized textbooks in Junior High School.
7. On a special problem-solving test at the end of grade nine, students studying Seeing Through Mathematics textbooks in Junior High School achieved equally well as the students studying the currently authorized textbooks in Junior High School.
8. Irrespective of the textbook/s studied, students of high and average ability obtained equal results at the end of grade nine on a special problem-solving test.
9. On a special problem-solving test at the end of grade nine, the low ability students studying Exploring Modern Mathematics textbooks achieved significantly higher scores than the students studying the three other mathematics programs. The low ability students studying these three other mathematics programs obtained equal results.
10. On the linear section of the special problem-solving test at the end of grade nine, students studying Exploring Modern Mathematics textbooks scored significantly higher than the students studying Seeing Through Mathematics textbooks. Students studying

Seeing Through Mathematics-Exploring Modern Mathematics textbooks scored significantly higher than the students studying Seeing Through Mathematics textbooks or the students studying the currently authorized textbooks. Students studying Seeing Through Mathematics textbooks scored as well as students studying the currently authorized textbooks, and the students studying the currently authorized textbooks scored as well as students studying Exploring Modern Mathematics textbooks.

11. While all groups of students at the average ability level obtained equal scores on the linear section of the special problem-solving test, the Seeing Through Mathematics-Exploring Modern Mathematics students at the high ability level scored significantly higher than the Seeing Through Mathematics students. At the low ability level the Exploring Modern Mathematics students scored significantly higher than the three other groups of low ability students.
12. On the proportional section of the special problem-solving test, all groups of students scored equally well.
13. On the quadratic section of the special problem-solving test, students studying Exploring Modern Mathematics textbooks scored significantly higher than the students studying Seeing Through Mathematics textbooks or the students studying the currently authorized textbooks. Students studying Exploring Modern Mathematics textbooks achieved equally well as the students studying Seeing Through Mathematics-Exploring Modern Mathematics textbooks and the Seeing Through Mathematics students scored

equally well as the students studying the currently authorized textbooks.

14. While all groups of students at the high and average ability levels scored equally well on the quadratic section of the special problem-solving test, the Exploring Modern Mathematics students at the low ability level scored significantly higher than the three other groups of low ability students.

Assuming the validity of the IOWA Test of Basic Skills, Form 1 (Grade Eight and Nine Problem-Solving subtest), one would conclude that the modern and traditional Junior High School mathematics textbooks were equally effective in developing students' traditional problem-solving proficiency. Assuming the validity of the Special Problem-Solving test, the Exploring Modern Mathematics and Seeing Through Mathematics- Exploring Modern Mathematics programs contribute more to the development of problem-solving proficiency of Junior High School students than either the Seeing Through Mathematics or the currently authorized conventional programs. Furthermore, students transferring from a grade eight Seeing Through Mathematics program to an Exploring Modern Mathematics program in grade nine reach the same level of problem-solving proficiency as the Exploring Modern Mathematics students and a higher level of proficiency than the Seeing Through Mathematics students. Therefore, on the basis of the performance of the students who transferred from the Seeing Through Mathematics program to the Exploring Modern Mathematics program at the beginning of grade nine, one could conclude that the transfer has contributed

considerably to their level of problem-solving proficiency.

When one considers the scores on the Special Problem-Solving test for each ability level, the Exploring Modern Mathematics program contributes more to the development of problem-solving proficiency at the low ability level. This is particularly true when mathematical concepts are involved in linear and quadratic relations. Assuming the validity of the Special Problem-Solving test, the findings indicate that the Exploring Modern Mathematics program is more effective than other programs in developing problem-solving proficiency of low ability students. However, all programs are equally effective in the development of problem-solving proficiency of the high and average ability students.

The investigator admits that there are some obvious limitations that must be placed on the interpretation of the findings of this study. First, the subjects included in this study were not a random sample of the Alberta grade nine population. However, Table IX, on page 55, reveals that a wide range of student scholastic ability levels is represented in the group. Second limitation might be the small number of students comprising the STM-EMM group. A third limitation might be the appropriateness of the Special Problem-Solving test as a measure of students' problem-solving proficiency. The reader is strongly urged to examine carefully the items in the final version of the test included in the Appendix.

IMPLICATIONS AND RECOMMENDATIONS

Perhaps the most apparent implication for future research is that the experiment reported herein should be replicated with tighter experimental controls. Since Exploring Modern Mathematics and Seeing Through Mathematics textbooks are presently authorized for use in Alberta schools at the grade seven level, it would be possible to establish procedures to insure random sampling of the Exploring Modern Mathematics and Seeing Through Mathematics groups of students. Furthermore, a teacher reaction to the problem-solving procedures recommended in the new mathematics programs should be studied. These studies should result in further improvement in mathematics instruction and curriculum content at the Junior High School level.

This study shows that students transferring from Seeing Through Mathematics program to an Exploring Modern Mathematics program achieve equally as well as, or better than, students who do not transfer. However, there is need for further study to determine the effect on students' problem-solving proficiency of transferring from Exploring Modern Mathematics program to a Seeing Through Mathematics program.

While a study such as the one described in this report provides an overall comparison of the relative contributions of each program to the development of students' problem-solving proficiency, there is a need for investigating the students' thinking patterns related to various problem situations presented by the different program materials. Such a study may help to explain why some of the Seeing Through

Mathematics classes completed only approximately two-thirds of the program during a period of three years.

There is need for a study of the relative contributions of the different mathematics programs to the development of logical deductions and valid generalizations at the Junior High School level.

Further research should be directed toward the standardization and eventual usage in Alberta schools of instruments designed to measure the various aspects of problem-solving proficiency.

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APPENDIX A

PROBLEM-SOLVING TEST

SECTION A.

1. A father is nine times as old as his son. If the son is n years old now, how old will the father be in 8 years?

   a. $8n + 9$
   b. $9(n + 8)$
   c. $9n + 8$
   d. $9n$

2. A rectangular field bordering a lake measures 150 feet long by 50 feet wide. A fence is to be put along the two short sides and one long side. How many feet of fence would be needed?

   a. 400
   b. 350
   c. 300
   d. 250

3. A merry-go-round has three rows of horses. The outer row is 6 feet farther out than the inner row. If you sit on an outside horse, how much farther do you ride in one turn of the merry-go-round than if you sit on an inside horse?

   a. 12π
   b. 6π
   c. $6 + \pi$
   d. 12

4. If $(8 + 2c)$ is less than $(12 + c)$, then c must be:

   a. any number
   b. any number greater than 4
   c. any number except 4
   d. any number less than 4

5. I have $(a + 2)$ dollars. If I spend one-half of that amount, I will have less than seven dollars left. What is the value of $\frac{a}{2}$?

   a. less than seven
   b. 5
   c. 6
   d. less than six

6. You are x years old and my age is twice yours. In three years the sum of our ages will be 27 years. Which mathematical sentence below represents this situation?

   a. $3x + 3 = 27$
   b. $3x + 6 = 27$
   c. $3x = 27$
   d. $9x = 27$

7. The current of a stream has a rate of x miles per hour. A motor boat goes downstream 10 miles per hour faster than the rate of the current. If the boat's speed downstream relative to land is 25 miles per hour, which mathematical statement below could be used to describe this situation?

   a. $\frac{x}{10} = 25$
   b. $x + 10 = 25$
   c. $10x = 25$
   d. $11x = 25$

8. If x dollars is the cost per yard of a certain cloth, and ten yards of cloth costs more than 12 dollars, which statement below could be used to describe this situation?

   a. $10x = 12$
   b. $10x$ is greater than 12
   c. $x + 10$ is greater than 12
   d. x is greater than 12

9. I have $3.90 in my pocket. If n represents the number of nickels I have and I have three times as many dimes as I have nickels and six more quarters than nickels, which mathematical statement below could be used to describe the situation?

   a. $n + 3n + n + 6 = 390$
   b. $n + 3n + 6n = 390$
   c. $5n + 10(3n) + 25(n+6) = 390$
   d. $5n + 10(3n) + 25(6n) = 390$

10. If five is added to twice a certain number n and the sum is divided by three, which is the correct form to express the situation?

   a. $2n + \frac{5}{3}$
   b. $2n + 5 = \frac{n}{3}$
   c. $\frac{2n + 5}{3}$
   d. $\frac{2n}{3} + 5$

11. State a number sentence for: If one number is five times another number n, the sum of the two numbers is 15 more than four times the smaller.

   a. $5n = 4n + 15$
   b. $n + 5n = 4n + 15$
   c. $n + 5n + 15 = 4n$
   d. $n + 5n = 15(4n)$

12. Two water pipes are bringing water into a reservoir. One pipe has a capacity of 100 gallons per minute, and the second 40 gallons per minute. If water flows from the first pipe for x minutes and from the second for y minutes, what is the total flow in gallons?

   a. $100x + 40y$
   b. 140
   c. $140x$
   d. $140(x + y)$

13. A man worked four days on a job and his son worked half as long. The father earned x dollars a day. The son's daily wage was $\frac{2}{5}$ that of his father. If they earned a total of $96, which mathematical statement describes the situation?

    a. $x + \frac{2}{5}x = 96$
    b. $4x + \frac{8x}{5} = 96$
    c. $4x + \frac{4x}{5} = 96$
    d. $4x + \frac{2x}{5} = 96$

14. The width of a rectangle is five inches less than its length L inches. Which mathematical statement describes the situation if the perimeter is 42 inches?

    a. $4L + 10 = 42$
    b. $2L - 5 = 42$
    c. $4L - 5 = 42$
    d. $4L - 10 = 42$

15. A square and an equilateral triangle have equal perimeters. A side of the triangle is 3.5 inches longer than a side of the square. If the length of the side of the square is s inches, which mathematical statement below expresses this condition?

    a. $4s = 3(3.5s)$
    b. $4s = 3(s - 3.5)$
    c. $4s = 3(s + 3.5)$
    d. $4s = 3s + 3.5$

16. John rides his bicycle at 10 miles per hour and Rudy rides his at 12 miles per hour. John starts from a point 5 miles east of the "0" sign-post and Rudy starts from a point 6 miles west of the "0" sign-post. If they start at the same time and both travel east, how many miles apart are they after they have travelled 3 hours?

    a. 5
    b. 1
    c. 6
    d. 0

17. John rides his bicycle at 10 miles per hour and Rudy rides his at 12 miles per hour. They both start from the "0" sign-post but John goes east while Rudy goes west. If John starts 15 minutes earlier than Rudy, how many miles apart are they one and one-half hours after John started?

    a. 35
    b. 30
    c. 15
    d. 7

18. One-half of a number n is three more than one-sixth of the same number. Which is the correct equation to find the number?

    a. $\frac{n}{2} + 3 = \frac{n}{6}$
    b. $\frac{n}{2} = \frac{3n}{6}$
    c. $\frac{n}{2} = 3 + \frac{n}{6}$
    d. $\frac{3n}{2} = \frac{n}{6}$

19. If the first positive integer is n and the sum of the three successive positive integers is 108, then the integers can be found by using which one of the following equations?

    a. $3n + 3 = 108$
    b. $3n = 108$
    c. $3n + 6 = 108$
    d. $6n = 108$

20. Terry has five hours at his disposal. He rides his bicycle without stopping m miles into the surrounding hills at the rate of 8 miles per hour and returns at the rate of 12 miles per hour. The correct equation to find the distance m miles he rides into the hills is:

    a. $8m + 12m = 5$
    b. $\frac{m}{8} + \frac{m}{12} = 5$
    c. $\frac{8m + 12m}{5} = 20$
    d. $\frac{8m}{12m} = 5$

21. In a class the average mark was 58. Three students were transferred into the class. Their marks of 56, 62 and 84 raised the class average to 59. What was the original number of students in the class?

    a. 32
    b. 28
    c. 29
    d. 25

22. A man 42 years of age has a daughter 8 years old. In y years he will be twice as old as his daughter. Which mathematical sentence below describes this situation?

    a. $42 + y = 16 + y$
    b. $42 + y = 16 + 2y$
    c. $y = 42 + 8$
    d. $42 + y = 16$

23. The greatest angle of a triangle measures 50 larger than the smallest angle x and 10 larger than the third angle. If the sum of the three angles is 180 which equation would you use to find x?

    a. $3x + 60 = 180$
    b. $3x + 110 = 180$
    c. $3x + 90 = 180$
    d. $61x = 180$

24. Printing press A can do a certain job in 6 hours. If press B works on the job at the same time, the two of them can do it in 2 hours. In how many hours can press B complete the job alone.

    a. 3
    b. 3.5
    c. 4
    d. 5

25. A man makes a trip of 300 miles at an average speed of 30 miles per hour and returns at an average speed of 20 miles per hour. What was the average speed in miles per hour for the entire trip?

    a. 6
    b. 25
    c. 24
    d. 12

26. Loriann bought 32 post cards. She used more than 13 of them, but still had more than 16 left. How many post cards did she use?

    a. 13 or 14.
    b. 15 or 16
    c. 14 or 15
    d. 16 or 17

PROBLEM-SOLVING TEST

SECTION B

1. Suppose that a sculptor wants to make a scale model of a monument by reducing each measurement to $\frac{1}{3}$ that of the original. The ratio of the volume of material that would be required for the scale model to that required for the monument would be:

   a. $\frac{1}{3}$
   b. $\frac{1}{27}$
   c. $\frac{1}{6}$
   d. $\frac{1}{9}$

2. If a speed of 38 miles per hour is equivalent to a speed of 33 knots, develop a statement which will enable to convert m miles per hour to k knots per hour.

   a. $38m = 33k$
   b. $\frac{k}{m} = \frac{38}{33}$
   c. $\frac{m}{k} = \frac{38}{33}$
   d. $\frac{38}{k} = \frac{33}{m}$

3. An increase of 20% in Mr. Jones' present income which is m dollars per month would give him an income of $300 per month. Which mathematical statement would enable you to find his correct present monthly income?

   a. $\frac{m}{300} = \frac{20}{100}$
   b. $\frac{300}{m} = \frac{120}{100}$
   c. $m + 20 = 300$
   d. $\frac{m}{300} = \frac{120}{100}$

4. Consider the following statements:

   A. My salary quadrupled is matched by Jack's salary tripled.
   B. There are four more bananas when the number of apples is increased by three.
   C. The perimeters of an equilateral triangle and a square have equal measures.
   D. Three times a certain number is equal to four times another number.

   Then $3a = 4b$ could represent

   a. all four situations above
   b. only A, C and D
   c. only B, C and D
   d. only A and D

5. A salesman earned a basic salary of \$80 a week. In addition, he received a commission of 3% of his total sales. During one week he earned \$116. What was the amount of his sales for the week?

   a. $116 \times \frac{100}{3}$
   b. $(116 - 80)\ \frac{100}{3}$
   c. $(116 + 80)\ \frac{100}{3}$
   d. $116 \times \frac{3}{100}$

6. The profits from a student show are to be given to two scholarship funds in the ratio of $\frac{2}{3}$. If the fund receiving the larger amount was given \$57, how much was given to the other fund?

   a. 95
   b. 38
   c. 19
   d. 76

7. In a shipment of 800 radios, one-twentieth of the radios were defective. What is the ratio of defective radios to non-defective radios in the shipment?

   a. $\frac{19}{20}$
   b. $\frac{1}{40}$
   c. $\frac{1}{20}$
   d. $\frac{1}{19}$

8. The numerator of the fraction $\frac{4}{7}$ is increased by such an amount that the value of the resulting fraction is equivalent to $\frac{27}{21}$. By what amount was the numerator increased?

   a. 23
   b. $6\frac{3}{4}$
   c. 5
   d. 3

9. If 92¢ is x percent of 80¢, then what mathematical statement would you use to find the value of x?

   a. $\frac{92}{80} = \frac{100}{x}$
   b. $\frac{92}{80} = \frac{x}{100}$
   c. $80x = 92$
   d. $\frac{80}{100} = \frac{x}{92}$

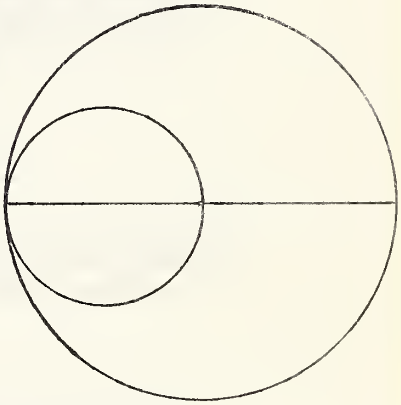
10. Suppose the population of an Alberta town was 1500 in 1950 and 1200 in 1960. If x represents the percent of decrease, then to find this percent of decrease you would use:

   a. $\frac{1200}{1500} = \frac{x}{100}$
   b. $\frac{300}{1500} = \frac{x}{100}$
   c. $\frac{1500}{1200} = \frac{x}{100}$
   d. $\frac{300}{1500} = \frac{100}{x}$

11. The rents in an apartment house are increased by 5%. A tenant who pays $80 per month for rent before the increase can compute the amount of the increase by using which mathematical statement?

   a. $\frac{80}{x} = \frac{5}{100}$
   b. $\frac{105}{100} = \frac{80}{x}$
   c. $\frac{x}{80} = \frac{5}{100}$
   d. $\frac{80}{100} = \frac{5}{x}$

12. The center of a larger circle lies on the smaller circle. The intersection of the two circles is a single point. This point and the centers of the two circles lie on one line. If the interior of the smaller circle is chosen as a unit of area measure, what would be the area measure of the region inside the larger circle but outside the smaller circle?



   a. 2
   b. 3
   c. 4
   d. 5

13. A rectangular sheet of metal for a machine is to have dimensions such that the ratio of the measure of the area to the measure of the perimeter is $\frac{1}{2}$. What equation would you use to find the width w feet, if the length of the rectangular sheet is 6 feet?

   a. $\frac{12 + 2w}{6w} = \frac{1}{2}$
   b. $\frac{6w}{12 + 2w} = \frac{1}{2}$
   c. $\frac{36w^2}{12 + 2w} = \frac{1}{2}$
   d. $\frac{6w}{12w} = \frac{1}{2}$

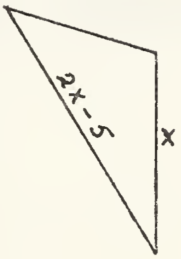
14. Fred's house is 40 feet long and 30 feet wide. What would be the dimensions of the floor plan in inches if he used a scale of $\frac{1}{4}$ inch : two feet?

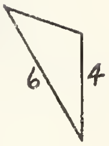
   a. 10 by $7\frac{1}{2}$
   b. 20 by 15
   c. 5 by $3\frac{3}{4}$
   d. 4 by 3

15. Two more than a number n divided by the number equals negative six divided by 3 less than the number. What mathematical sentence describes the relationship?

    a. $\frac{2 + n}{2} = \frac{6}{n - 3}$
    b. $n + \frac{2}{n} = {}^{-}6$
    c. $\frac{2}{n} = \frac{^{-}6}{^{-}3n}$
    d. $\frac{2 + n}{n} = \frac{^{-}6}{n - 3}$

16. The sides of lengths x and $2x - 5$ of the first triangle shown have the same ratio as the sides of lengths 4 and 6 respectively of the second triangle. How long is the longest side of the first triangle?





    a. 5
    b. 10
    c. 15
    d. 25

17. Barry weighs 80 pounds and Douglas weighs 100 pounds. They balance on a teeterboard that is 9 feet long. Each sits at an end of the board. How many feet from the point of balance is Barry sitting?

    a. 3.5
    b. 4
    c. 4.5
    d. 5

18. When 4 cubic inches of water freeze, 4.4 cubic inches of ice are formed. How many cubic inches of water must freeze to form 66 cubic inches of ice?

    a. 72.6
    b. 60
    c. 59.4
    d. 66.4

19. A wheel that has a circumference of 9 inches must revolve 72 times to cover a certain distance. How many times must a wheel that has a circumference of 4 inches revolve to cover the same distance?

    a. 81
    b. 32
    c. 18
    d. 162

20. Robert buys a new camera for \$126 and agrees to pay for it in x months. What sentence would you use to find the monthly payment \$ y?

    a. $x + y = 126$
    b. $\frac{126}{x} = \frac{1}{y}$
    c. $\frac{1}{x} = \frac{126}{y}$
    d. $\frac{1}{x} = \frac{y}{126}$

21. On a floor plan of the house, 2 inches represents 9 feet of actual distance. How many inches on the floor plan will represent an actual distance of 12 feet?

a. 3
b. $2\frac{3}{4}$
c. $2\frac{2}{3}$
d. $2\frac{3}{8}$

22. The length of the base of a parallelogram is 15 feet. This base is 2.5 times as long as the altitude. What is the area in square feet?

a. 37.5
b. 75
c. 90
d. 45

SECTION C.

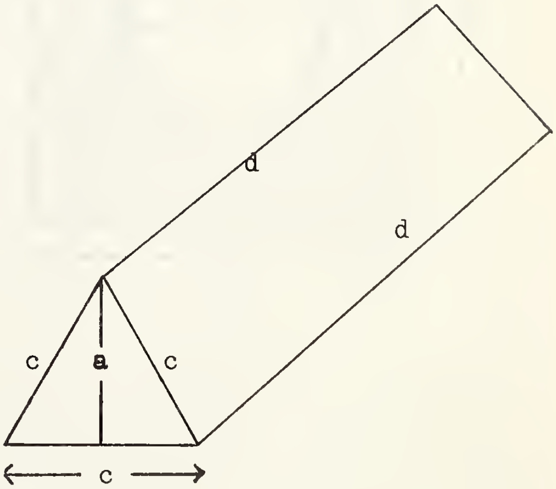
1. One end of a 50-foot wire is attached to the top of a vertical telephone pole. The wire is pulled taut and the lower end is attached to a concrete block on the ground. This block is 30 feet from the base of the telephone pole at ground level. What is the height of the pole in feet?

a. $50 + 30$
b. $\sqrt{50^2 + 30^2}$
c. $\sqrt{50^2 - 30^2}$
d. $\sqrt{50^2} - \sqrt{30^2}$

2. The perimeter of a square is less than ten inches. What do you know about the area of this square?

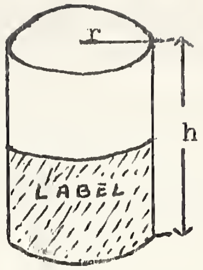
a. area is less than 10 square inches
b. area is less than 100 square inches
c. area is less than 5 square inches
d. area is less than $6\frac{1}{4}$ square inches

3. A pup tent in the shape of a triangular prism is d feet long. The measurements in feet of one end are given in the diagram. This pup tent has canvas ends and floor. How many square feet of canvas would you use to construct it? Make no allowances for seams, doors or windows.



a. ac + dc
b. 2ac + 2cd
c. ac + 3cd
d. 2ac + 3cd

4. If the label covers only one-half of the curved surface of a tin can shown on the right, how would you represent the total surface area of the can that is NOT covered by the label?



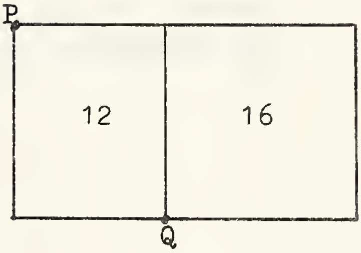
a. πrh

b. $2\pi r^2 + 2\pi rh$

c. $2\pi r^2 + \pi rh$

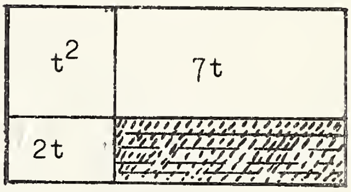
d. $2\pi r^2 + \frac{h}{2}$

5. The rectangle illustrated consists of a square of area 16 and a rectangle of area 12 square units. What is the measure of the distance from P to Q?



a. 4
b. 5
c. 6
d. 7

6. Given the areas of the unshaded sections of the rectangle, what is the area of the shaded section?

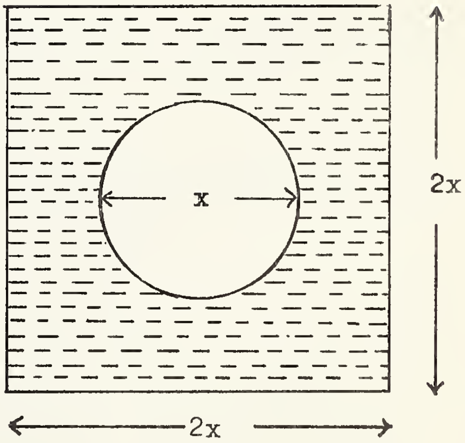


a. $14t^2$

b. $3\frac{1}{2}t^2$

c. 14
d. 7

7. What mathematical statement would you use to find the shaded area of the figure on the right?



a. $2x^2 - \frac{\pi x^2}{2}$

b. $4x^2 - \pi x^2$

c. $4x^2 - \frac{\pi x^2}{2}$

d. $4x^2 - \frac{\pi x^2}{4}$

8. An airplane is headed south in a wind whose velocity is 45 miles per hour east. The speed of the airplane, with no wind is 225 miles per hour. Which sentence expresses the resultant velocity v miles per hour of the airplane?

   a. $v = \sqrt{225^2 - 45^2}$
   b. $v = 225^2 - 45^2$
   c. $v = \sqrt{225^2 + 45^2}$
   d. $v = 225 + 45$

9. The square of a number is 91 more than 6 times the number. Which statement describes this situation if the number is represented by n?

   a. $\sqrt{n} = 6n + 91$
   b. $n^2 = 6n - 91$
   c. $n^2 = 91 - 6n$
   d. $n^2 = 91 + 6n$

10. Find the width of a rectangle whose perimeter is 28 feet and whose area is 24 square feet.

   a. 2
   b. 3
   c. 4
   d. 6

11. Using the formula $r = \sqrt{\frac{3s}{k}}$, find a formula for k in terms of r and s.

   a. $k = \sqrt{\frac{3s}{r}}$
   b. $k = \frac{3s}{r^2}$
   c. $k = \frac{3s}{r}$
   d. $k = \frac{\sqrt{3s}}{r}$

ITEM DISCRIMINATION AND DIFFICULTY
INDICES FOR PILOT STUDY VERSION OF SPS TEST

SECTION A Linear Relations				SECTION B Proportional Relations				SECTION C Quadratic Relations			
NP	r	d	NF	NP	r	d	NF	NP	r	d	NF
1	.285	.914		1	.375	.140		1	.461	.380	25
2	-.078	.903		2	.352	.366		2	.548	.215	
3	.481	.258	26	3	.532	.323	29	3	.368	.323	30
4	.322	.731		4	.573	.516	22	4	.313	.129	
5	.676	.312	21	5	.513	.602	12	5	.350	.412	20
6	.371	.323		6	.447	.570	17	6	.426	.451	10
7	.457	.785	1	7	.252	.151		7	-.102	.108	
8	.519	.763		8	.423	.290		8	.112	.118	
9	.285	.376		9	.306	.581		9	.316	.538	5
10	.651	.849		10	.551	.398	27	10	.585	.398	15
11	.567	.559	11	11	.496	.323		11	.288	.242	
12	.285	.914		12	.494	.441	24				
13	.636	.247		13	.463	.215					
14	613	.548	13	14	.484	.613	9				
15	.558	.624	3	15	.563	.860	2				
16	.505	.505	16	16	.087	.366					
17	.620	.376		17	.393	.387					
18	.464	.591	6	18	.441	.624	7				
19	.124	.473		19	.318	.638	4				
20	.489	.204	28	20	.739	.172					
21	.262	.204		21	.675	.581	14				
22	.489	.301	23	22	.663	.538	19				
23	.450	.366	18								
24	-.094	.333									
25	.526	.054									
26	.497	.581	8								

NP - Item number of the pilot study version of the test

r - biserial correlation

d - difficulty index

NF - Item number of the final version of the test

APPENDIX B

INSTRUCTIONS TO STUDENTS

This is a 45-minute test containing 30 multiple-choice questions. Read each question carefully. (EMPHASIZE CHANGE) Do NOT do your rough work in the test booklet. Use the scratch paper provided. Be sure to mark your answer choice for each question on the answer sheet provided.

Here is a sample question:

44. A father is nine times as old as his son. If the son is n years old now, how old will the father be in 8 years?

A. $8n + 9$

B. $9n + 8$

C. $9(n + 8)$

D. $9n$

For practice, find item 44 on the answer sheet and use your H.B. pencil to make a mark about 1/32 of an inch wide running the full length of the guideline under the letter you have chosen to indicate your answer.

This is what you will find at item 44 on the answer sheet.

	A	B	C	D
44.	___	___	___	___

The right answer is ....B. Make a mark about 1/32 of an inch wide running the full length of the guideline under the letter B. Your mark should appear similar to the one in the example on top of your answer sheet.

Mark only one answer for each question. Make all your answer marks black and about 1/32 of an inch wide running the full length of the guideline. DO NOT MAKE A MARK LONGER THAN THE GUIDELINE. The most satisfactory marks are made by using a sharp H.B. pencil and stroking over the guideline two or three times.

If you change your mind about an answer, erase the unwanted answer completely.

Be careful not to place any marks of any kind among the timing lines (the black lines in the right-hand margin of the answer sheet). Watch particularly so as not to run the personal information such as the name of the school and the name of the test into the timing marks.

DO NOT FOLD AN ANSWER SHEET NOR BEND ANY OF THE CORNERS

1. The current of a stream has a rate of x miles per hour. A motor boat goes downstream 10 miles per hour faster than the rate of the current. If the boat's speed downstream relative to land is 25 miles per hour, which mathematical statement below could be used to describe this situation?

   A. $\frac{x}{10} = 25$

   B. $x + 10 = 25$

   C. $10x = 25$

   D. $x - 10 = 25$

2. Two more than a number n divided by the number equals negative six divided by 3 less than the number. Which mathematical sentence describes the relationship?

   A. $\frac{2 + n}{2} = \frac{6}{n - 3}$

   B. $n + \frac{2}{n} = {}^{-}6$

   C. $\frac{2}{n} = \frac{^{-}6}{^{-}3n}$

   D. $\frac{2 + n}{n} = \frac{^{-}6}{n - 3}$

3. A square and an equilateral triangle have equal perimeters. A side of the triangle is 3.5 inches longer than a side of the square. If the length of the side of the square is s inches, which mathematical statement below expresses this condition?

   A. $4s = 3\ (3.5s)$

   B. $4s = 3\ (s - 3.5)$

   C. $4s = 3\ (s + 3.5)$

   D. $4s = 3s + 3.5$

4. A wheel that has a circumference of 9 inches must revolve 72 times to cover a certain distance. How many times must a wheel that has a circumference of 4 inches revolve to cover the same distance?

   A. 81

   B. 32

   C. 18

   D. 162

5. The square of a number is 91 more than 6 times the number. Which statement describes this situation if the number is represented by n?

A. $\sqrt{n} = 6n + 91$

B. $n^2 = 6n - 91$

C. $n^2 = 91 - 6n$

D. $n^2 = 91 + 6n$

6. One-half of a number n is three more than one-sixth of the same number. Which is the correct equation to find the number?

A. $\frac{n}{2} + 3 = \frac{n}{6}$

B. $\frac{n}{2} = \frac{3n}{6}$

C. $\frac{n}{2} = 3 + \frac{n}{6}$

D. $\frac{3n}{2} = \frac{n}{6}$

7. When 4 cubic inches of water freeze, 4.4 cubic inches of ice are formed. How many cubic inches of water must freeze to form 66 cubic inches of ice?

A. 72.6
B. 60
C. 59.4
D. 66.4

8. Loriann bought 32 post cards. She used more than 13 of them, but still had more than 16 left. How many post cards did she use?

A. 13 or 14
B. 15 or 16
C. 14 or 15
D. 16 or 17

9. Fred's house is 40 feet long and 30 feet wide. What would be the dimensions of the floor plan in inches if he used a scale of $\frac{1}{4}$ inch: two feet?

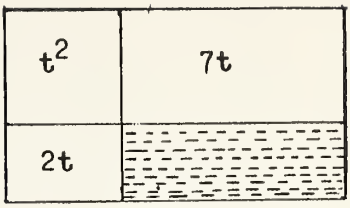
A. 5 by $3\frac{3}{4}$

B. 20 by 15

C. 10 by $7\frac{1}{2}$

D. 4 by 3

10. Given the areas of the unshaded sections of the rectangle, what is the area of the shaded section?



A. $14t^2$

B. $3\frac{1}{2}t^2$

C. 14

D. 7

11. State a number sentence for: If one number is five times another number n, the sum of the two numbers is 15 more than four times the smaller.

A. $5n = 4n + 15$

B. $n + 5n = 4n + 15$

C. $n + 5n + 15 = 4n$

D. $n + 5n = 15\ (4n)$

12. A salesman earned a basic salary of $80 a week. In addition, he received a commission of 3% of his total sales. During one week he earned $116. What was the amount of his sales for the week?

A. $(116 - 80)\ \frac{100}{3}$

B. $116 \times \frac{100}{3}$

C. $(116 + 80)\ \frac{100}{3}$

D. $116 \times \frac{3}{100}$

13. The width of a rectangle is five inches less than its length, L inches. Which mathematical statement describes the situation if the perimeter is 42 inches?

A. $4\,L + 10 = 42$

B. $2\,L - 5 = 42$

C. $4\,L - 5 = 42$

D. $4\,L - 10 = 42$

14. On a floor plan of the house, 2 inches represents 9 feet of actual distance. How many inches on the floor plan will represent an actual distance of 12 feet?

   A. 3

   B. $2\frac{3}{4}$

   C. $2\frac{2}{3}$

   D. $2\frac{3}{8}$

15. Find the width of a rectangle whose perimeter is 28 feet and whose area is 24 square feet.

   A. 2
   B. 3
   C. 4
   D. 6

16. John rides his bicycle at 10 miles per hour and Rudy rides his at 12 miles per hour. John starts from a point 5 miles east of the "O" sign-post and Rudy starts from a point 6 miles west of the "O" sign-post. If they start at the same time and both travel east, how many miles apart are they after they have travelled 3 hours?

   A. 5
   B. 1
   C. 6
   D. 0

17. The profits from a student show are to be given to two scholarship funds in the ratio of $\frac{2}{3}$. If the fund receiving the larger amount was given $57, how much was given to the other fund?

   A. 95
   B. 38
   C. 19
   D. 76

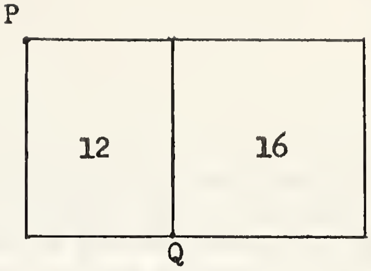
18. The greatest angle of a triangle measures 50 larger than the smallest angle x and 10 larger than the third angle. If the sum of the three angles is 180, which equation would you use to find x?

   A. $3x + 90 = 180$
   B. $3x + 110 = 180$
   C. $3x + 60 = 180$
   D. $61x = 180$

19. The length of the base of a parallelogram is 15 feet. This base is 2.5 times as long as the altitude. What is the area in square feet?

   A. 37.5
   B. 75
   C. 90
   D. 45

20. The rectangle illustrated consists of a square of area 16 and a rectangle of area 12 square units. What is the measure of the distance from P to Q?



   A. 4
   B. 5
   C. 6
   D. 7

21. I have (a + 2) dollars. If I spend one-half of that amount, I will have less than seven dollars left. What is the value of $\frac{a}{2}$?

   A. less than seven
   B. 5
   C. 6
   D. less than six

22. Consider the following statements:

   a. My salary quadrupled is matched by Jack's salary tripled.
   b. There are four more bananas when the number of apples is increased by three.
   c. The perimeters of an equilateral triangle and a square have equal measures.
   d. Three times a certain number is equal to four times another number.

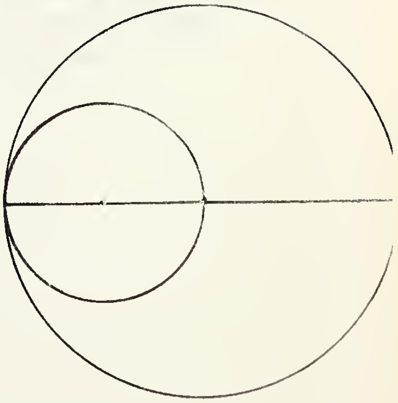
   Then $3a = 4b$ could represent

   A. all four situations above
   B. only a, c and d
   C. only b, c and d
   D. only a and d

23. A man 42 years of age has a daughter 8 years old. In y years he will be twice as old as his daughter. Which mathematical sentence below describes this situation?

   A. $42 + y = 16 + y$
   B. $42 + y = 16$
   C. $y = 42 + 8$
   D. $42 + y = 16 + 2y$

24. The center of a larger circle lies on the smaller circle. The intersection of the two circles is a single point. This point and the centers of the two circles lie on one line. If the interior of the smaller circle is chosen as a unit of area measure, what would be the area measure of the region inside the larger circle but outside the smaller circle?



   A. 2
   B. 3
   C. 4
   D. 5

25. One end of a 50-foot wire is attached to the top of a vertical telephone pole. The wire is pulled taut and the lower end is attached to a concrete block on the ground. This block is 30 feet from the base of the telephone pole at ground level. What is the height of the pole in feet?

    A. $50 + 30$
    B. $\sqrt{50^2 + 30^2}$
    C. $\sqrt{50^2 - 30^2}$
    D. $\sqrt{50^2} - \sqrt{30^2}$

26. A merry-go-round has three rows of horses. The outer row is 6 feet farther out than the inner row. If you sit on an outside horse, how much farther do you ride in one turn of the merry-go-round than if you sit on an inside horse?

    A. 12π
    B. 6π
    C. $6 + \pi$
    D. 12

27. Suppose the population of an Alberta town was 1500 in 1950 and 1200 in 1960. If x represents the percent of decrease, then to find this percent of decrease you would use:

    A. $\frac{1200}{1500} = \frac{x}{100}$
    B. $\frac{300}{1500} = \frac{x}{100}$
    C. $\frac{1500}{1200} = \frac{x}{100}$
    D. $\frac{300}{1500} = \frac{100}{x}$

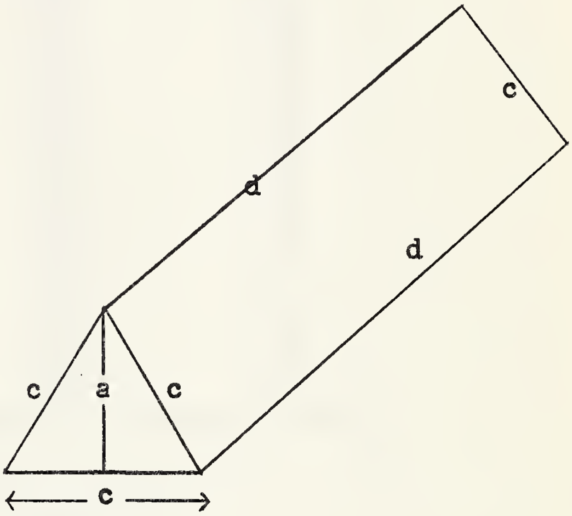
28. Terry has five hours at his disposal. He rides his bicycle without stopping m miles into the surrounding hills at the rate of 8 miles per hour and returns at the rate of 12 miles per hour. The correct equation to find the distance m miles he rides into the hills is:

    A. $8m + 12m = 5$
    B. $\frac{8m}{12m} = 5$
    C. $\frac{8m + 12m}{5} = 20$
    D. $\frac{m}{8} + \frac{m}{12} = 5$

29. An increase of 20% in Mr. Jones' present income which is m dollars per month would give him an income of $300 per month. Which mathematical statement would enable you to find his correct present monthly income?

A. $\frac{300}{m} = \frac{120}{100}$
B. $\frac{m}{300} = \frac{20}{100}$
C. $m + 20 = 300$
D. $\frac{m}{300} = \frac{120}{100}$

30. A pup tent in the shape of a triangular prism is d feet long. The measurements in feet of one end are given in the diagram. This pup tent has canvas ends and floor. How many square feet of canvas would you use to construct it? Make no allowances for seams, doors or windows.



A. ac + dc
B. 2ac + 2cd
C. ac + 3cd
D. 2ac + 3cd

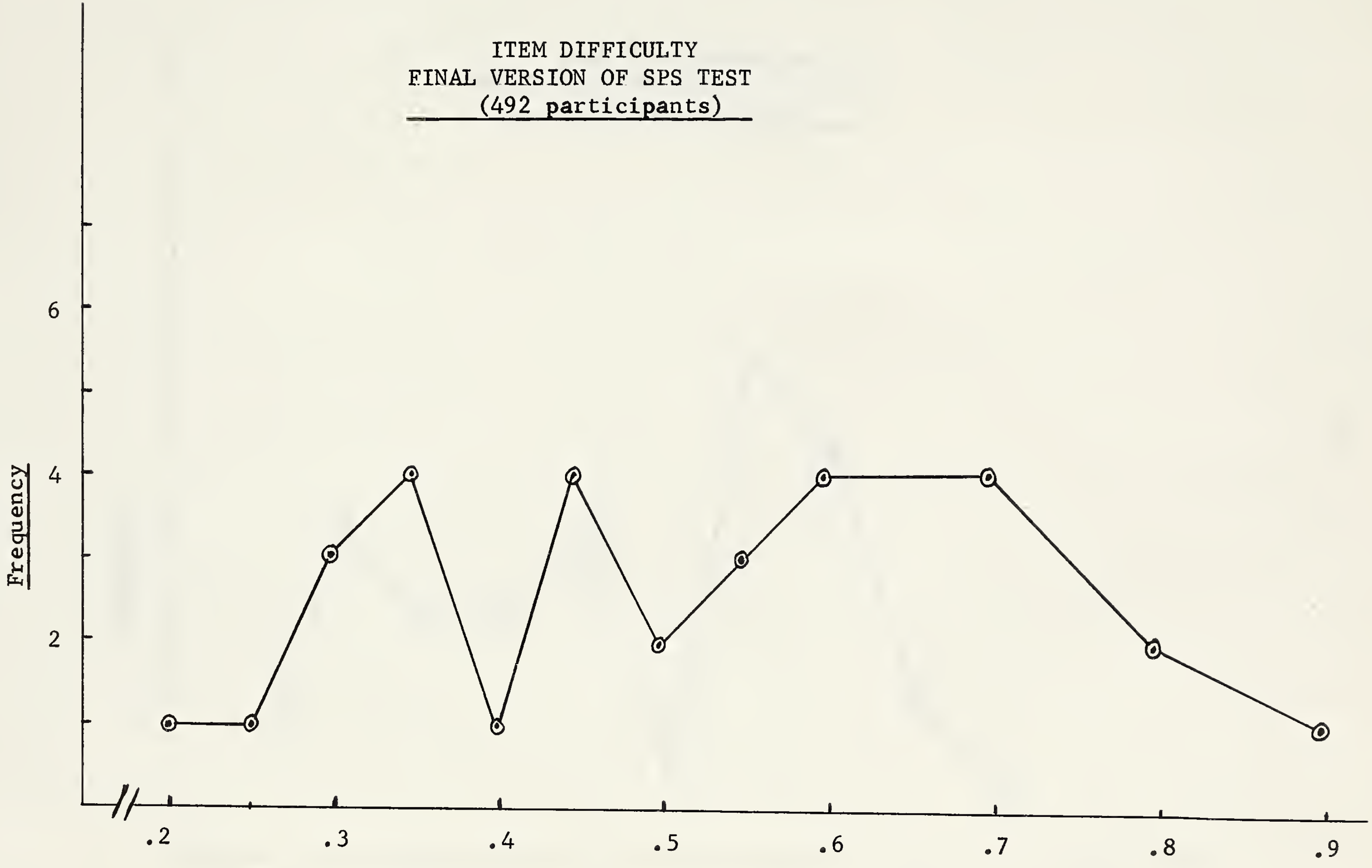
ITEM DISCRIMINATION AND DIFFICULTY INDICES
FOR FINAL VERSION OF SPS TEST
(492 participants)

NF	r	d	NF	r	d
1	.589	.709	16	.616	.439
2	.356	.876	17	.458	.805
3	.579	.587	18	.371	.293
4	.374	.480	19	.631	.602
5	.510	.677	20	.704	.440
6	.479	.693	21	.367	.363
7	.580	.720	22	.577	.533
8	.481	.587	23	.405	.327
9	.636	.559	24	.314	.374
10	.449	.211	25	.474	.339
11	.593	.557	26	.414	.230
12	.520	.593	27	.643	.427
13	.650	.516	28	.355	.381
14	.529	.780	29	.670	.302
15	.732	.470	30	.689	.398

NF - Item number of the final version of the test

r - biserial correlation

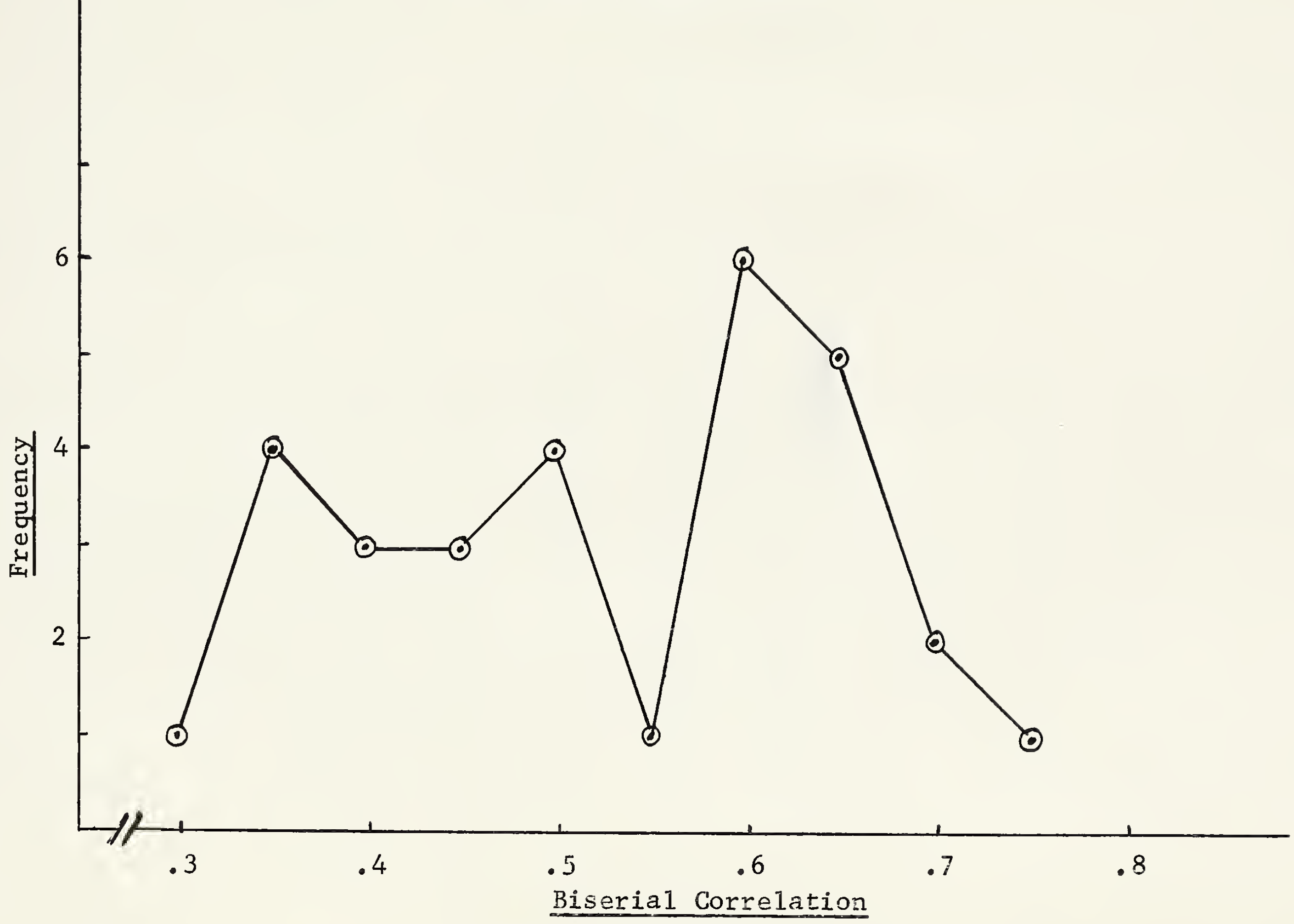
d - difficulty index



ITEM DIFFICULTY
FINAL VERSION OF SPS TEST
(492 participants)

Index of Difficulty
Range from .211 to .876
Average .509

ITEM DISCRIMINATION
FINAL VERSION OF THE SPS TEST
(492 participants)



range from .314 to .732

average .525

APPENDIX C

RAW SCORES FOR ALL STUDENTS

The scores obtained by students are listed on the following pages according to the mathematics programs they studied. The symbols heading the columns are to be interpreted as follows:

Column 1 APS - 63 - September, 1963, IOWA Arithmetic Problem Solving Score

Column 2 APS - 64 - March, 1964, IOWA Arithmetic Problem Solving Score

Column 3 APS - 65 - May, 1965, IOWA Arithmetic Problem Solving Score

Column 4 V - 63 - September, 1963, SCAT Verbal Ability Score

Column 5 Q - 63 - September, 1963, SCAT Quantitative Ability Score

Column 6 V + Q - 63 - September, 1963, SCAT Total Score (Verbal and Quantitative)

Column 7 SPS - Q - May, 1965, Special Problem, Solving Score on Quadratic Relations

Column 8 SPS - P - May, 1965, Special Problem Solving Score on Proportional Relation

Column 9 SPS - L - May, 1965, Special Problem Solving Score on Linear Relations

Column 10 SPS - May, 1965, Special Problem Solving Total Score

Column 11 ID - Identification Number

SEEING THROUGH MATHEMATICS STUDENTS

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
7	11	13	33	25	58	2	6	4	12	1
6	9	19	53	27	80	2	3	4	9	2
11	16	14	45	35	80	1	7	6	14	3
12	13	17	54	42	96	1	4	2	7	4
8	16	25	57	49	106	5	10	7	22	5
10	15	14	44	31	75	1	7	2	10	6
13	9	16	50	36	86	2	6	5	13	7
7	12	13	55	35	90	0	8	5	13	8
14	17	26	57	47	104	3	9	6	18	9
4	7	0	45	32	77	1	3	3	7	10
12	12	18	45	33	78	0	3	2	5	11
8	9	22	33	44	77	0	5	1	6	12
11	16	22	43	31	74	2	3	3	8	13
10	9	15	42	41	83	2	5	2	9	14
7	13	19	54	42	96	0	6	4	10	15
12	14	20	53	48	101	1	8	2	11	16
15	17	21	50	43	93	4	7	7	18	17
14	19	19	56	45	101	5	10	8	23	18
11	17	17	51	40	91	1	5	2	8	19
20	18	24	53	45	98	0	5	3	8	20
11	13	14	39	32	71	1	3	3	7	21
14	8	12	46	33	79	1	3	4	8	22
13	19	29	54	48	102	3	8	7	18	23
6	8	10	53	26	79	1	4	3	8	24
15	23	24	54	45	99	3	9	2	14	25
17	14	22	55	44	99	2	7	6	15	26
11	9	13	52	45	97	0	2	0	2	27
13	16	6	48	34	82	0	5	2	7	28
10	14	14	44	35	79	2	6	7	15	29
25	23	19	47	42	89	4	8	4	16	30
12	15	22	51	40	91	3	8	6	17	31
21	23	29	57	46	103	4	9	10	23	32
12	20	17	45	35	80	1	4	5	10	33
19	16	19	49	42	91	4	6	4	14	34
14	14	15	46	40	86	0	4	5	9	35
7	17	18	47	39	86	3	6	6	15	36
7	5	10	37	26	63	0	3	1	4	37
8	9	7	32	34	66	1	4	3	8	38
17	23	20	51	45	96	2	7	6	15	39
7	9	9	40	26	66	2	1	6	9	40
15	19	22	55	40	95	2	7	4	13	41
18	17	20	54	40	94	1	9	6	16	42
10	17	21	37	40	77	3	7	1	11	43
10	8	8	38	25	63	0	3	3	6	44

SEEING THROUGH MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
19	19	30	57	47	104	5	7	9	21	45
11	16	17	47	33	80	2	6	3	11	46
12	14	16	38	29	67	0	2	3	5	47
13	17	22	51	39	90	3	6	3	12	48
18	17	18	43	35	78	1	8	5	14	49
10	16	19	53	31	84	2	6	5	13	50
10	12	14	49	34	83	2	6	4	12	51
11	16	21	52	32	84	0	6	2	8	52
17	17	18	45	31	76	1	3	3	7	53
13	21	19	49	44	93	1	7	2	10	54
5	9	10	50	29	79	0	1	0	1	55
11	11	16	53	38	91	2	7	5	14	56
10	13	20	56	35	91	2	7	2	12	57
6	12	15	48	27	75	0	3	5	8	58
11	11	17	58	37	95	4	6	6	16	59
16	18	21	50	43	93	5	11	6	22	60
18	20	27	58	45	103	4	10	8	22	61
19	16	29	56	44	100	3	10	9	22	62
8	14	17	38	37	75	1	6	2	9	63
14	18	26	58	47	105	4	10	8	22	64
11	11	13	50	29	79	1	2	2	5	65
6	10	11	45	39	84	0	5	2	8	66
22	21	30	56	48	104	4	9	4	17	67
6	12	13	45	35	80	0	4	1	5	68
15	13	18	45	38	83	0	4	6	10	69
18	14	26	54	49	103	2	8	5	15	70
14	15	26	47	48	95	2	8	3	13	71
12	17	21	56	48	104	2	7	6	15	72
17	18	19	48	40	88	2	8	4	14	73
19	17	25	54	47	101	3	11	7	21	74
11	12	22	56	37	93	2	8	3	13	75
13	17	17	54	40	94	2	9	9	18	76
6	9	12	34	42	76	1	3	1	5	77
10	9	22	49	44	93	1	8	5	14	78
15	15	16	42	42	84	2	4	4	10	79
14	19	28	56	43	99	2	9	7	18	80
11	19	24	48	48	96	3	11	9	23	81
6	8	10	48	24	72	1	0	3	7	82
11	17	16	51	38	89	0	6	4	10	83
11	10	19	55	33	88	1	7	0	9	84
13	13	17	49	38	87	0	5	5	10	85
11	19	28	59	48	107	3	8	8	19	86
20	23	29	54	48	102	6	12	10	28	87
6	13	14	31	30	61	0	7	4	11	88

SEEING THROUGH MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
12	21	24	52	39	91	0	6	2	8	89
13	15	15	49	33	82	2	3	2	7	90
8	10	15	48	33	81	2	4	4	10	91
20	23	23	51	39	90	4	5	7	16	100
11	15	21	41	33	74	1	5	0	10	101
7	14	20	41	40	81	4	8	7	19	102
12	11	19	50	35	85	1	4	3	8	103
14	10	19	37	37	74	2	4	7	13	104
12	13	26	41	38	79	1	4	1	6	105
10	19	25	51	43	94	4	9	7	20	106
12	14	14	41	36	77	2	4	6	12	107
14	21	24	47	39	86	2	4	4	10	108
11	16	21	49	41	90	2	5	9	16	109
14	18	23	49	42	91	2	7	4	13	110
13	17	23	43	30	73	3	8	4	15	111
24	21	34	57	50	107	5	11	8	24	112
18	18	29	55	40	95	5	11	7	23	113
24	26	30	58	48	106	6	11	11	28	114
22	25	29	57	47	104	5	12	6	23	115
16	19	27	50	44	94	4	8	6	18	116
20	20	28	56	41	97	6	10	9	25	117
16	17	22	38	40	78	1	7	4	12	118
9	8	25	39	38	77	3	7	5	15	119
9	16	17	54	28	82	2	3	3	8	120
17	21	29	53	46	99	3	9	9	21	121
19	24	31	53	48	101	6	8	6	20	122
10	17	24	41	34	75	2	8	2	12	123
14	17	20	41	39	80	1	7	4	12	124
24	26	33	46	47	93	5	11	8	24	125
15	15	24	49	42	92	2	5	4	11	126
12	17	23	51	44	95	1	8	4	13	127
16	22	29	51	44	95	2	9	7	18	128
11	19	23	58	46	104	2	6	8	16	129
15	17	18	50	43	93	0	6	3	9	130
18	26	28	49	46	95	5	10	10	25	131
10	21	26	53	42	95	2	8	7	17	132
22	28	28	49	49	98	3	11	6	29	133
18	28	30	54	50	104	3	8	4	15	134
21	23	23	49	44	93	3	7	5	15	135
11	22	19	45	46	91	2	7	8	17	136
11	20	25	50	34	84	2	4	3	9	137
11	15	18	53	38	91	2	4	4	10	138
13	19	20	53	35	88	1	5	3	9	139
12	18	25	35	46	81	3	10	6	19	140

SEEING THROUGH MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
15	22	25	53	45	98	3	7	6	16	141
19	18	24	49	43	92	1	8	7	16	142
14	13	21	50	45	95	2	5	4	11	143
23	22	32	52	48	100	5	9	8	22	144
22	28	29	52	47	99	3	10	8	21	145
8	17	20	55	42	97	2	8	4	14	146
19	18	29	54	41	95	1	7	3	11	147
15	27	23	59	47	106	5	11	10	26	148
12	22	25	41	40	81	2	5	4	11	149
16	14	24	34	45	79	1	9	7	17	150
13	17	28	49	37	86	1	7	2	10	151
21	20	19	44	38	82	3	9	5	17	152
17	21	20	57	43	100	2	10	8	20	153
13	22	22	47	41	88	1	1	3	5	154
16	20	15	41	40	81	2	6	5	13	155
21	23	27	52	46	98	0	7	7	14	156
13	22	23	51	41	92	2	8	6	16	157
23	25	30	55	43	98	2	8	5	15	158
15	18	30	55	40	95	2	8	4	14	159
17	22	26	51	46	97	2	9	8	19	160
17	19	29	55	44	99	5	9	12	26	161
24	27	28	53	50	103	2	10	5	17	162
22	26	29	50	47	97	4	8	8	20	163
16	26	21	46	42	88	3	8	8	19	164
20	24	25	51	46	97	1	6	2	9	165
24	30	31	58	49	107	4	12	10	26	166
20	21	22	56	46	102	3	9	7	19	167
19	25	25	57	48	105	5	8	10	23	168
19	24	26	56	46	102	2	9	9	20	169
25	28	34	57	50	107	5	11	11	27	170
14	22	25	56	44	100	2	6	8	16	171
12	23	22	47	38	85	2	5	6	13	172
15	23	24	50	46	95	3	8	8	19	173
26	28	28	52	46	98	5	9	5	19	174
22	26	30	53	48	101	2	8	7	17	175
13	25	32	58	47	105	6	12	12	30	176
18	28	26	56	47	103	5	9	6	20	177
24	30	31	56	45	101	2	8	7	17	178
13	15	26	46	40	90	3	8	6	17	179
19	20	26	55	47	102	4	9	9	22	180
17	18	29	46	46	92	2	8	2	12	181
17	26	32	54	47	101	4	7	7	18	182
15	21	20	53	45	98	3	8	6	17	183
21	19	26	56	43	99	2	9	5	16	184

SEEING THROUGH MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
16	15	21	39	42	81	1	5	0	6	200
15	14	16	46	33	79	1	6	4	11	201
12	19	20	44	40	84	1	5	2	8	202
14	15	25	42	32	74	5	8	3	16	203
19	25	29	53	46	99	3	9	6	18	204
10	15	29	37	36	73	2	6	5	13	205
16	21	20	54	40	94	1	10	6	17	206
20	26	25	47	45	92	4	8	6	18	207
12	16	18	42	40	82	0	10	8	18	208
11	19	22	48	38	86	3	9	7	19	209
17	19	24	42	32	74	1	6	5	12	210
9	16	17	48	42	90	2	7	3	12	211
9	12	16	49	32	81	0	6	3	9	212
12	15	18	55	42	97	3	6	6	15	213
16	12	24	50	37	87	3	9	4	16	214
7	16	20	35	27	62	0	5	3	8	215
15	19	24	55	47	102	1	7	6	14	216
18	11	21	38	31	71	1	4	4	9	217
19	16	21	37	41	78	0	4	3	7	218
10	14	12	50	34	84	1	4	3	8	219
11	17	19	45	40	85	1	5	3	9	220

EXPLORING MODERN MATHEMATICS STUDENTS

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
19	25	23	47	48	95	4	7	9	20	300
11	9	22	54	31	85	2	7	6	15	301
9	9	14	54	33	87	1	8	5	14	302
18	21	20	41	39	80	1	7	5	13	303
17	13	19	50	33	83	1	8	5	14	304
18	10	11	56	38	94	1	3	5	9	305
8	17	15	58	33	91	2	9	6	17	306
17	18	23	50	45	95	3	10	8	21	307
19	21	24	53	46	99	1	8	8	17	308
8	10	17	43	39	82	3	8	4	15	309
14	13	22	57	47	104	5	8	5	18	310
11	12	18	48	42	90	3	6	4	13	311
11	8	15	56	36	82	2	6	7	15	312
15	22	26	58	46	104	1	9	9	19	313
15	16	23	52	44	96	2	5	8	15	314

EXPLORING MODERN MATHEMATICS STUDENTS (cont'd)

	IOWA APS SCORES		SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
10	20	24	57	32	89	5	9	10	24	315
14	17	23	51	44	95	4	8	7	19	316
14	15	15	52	42	94	5	10	6	21	317
20	27	29	55	44	99	2	9	9	20	318
11	14	20	53	39	92	1	8	5	14	319
23	23	25	51	38	89	0	5	4	9	320
11	12	15	53	33	86	1	8	4	13	321
13	13	25	50	36	86	0	8	7	15	322
17	20	26	56	41	97	3	9	6	18	323
10	17	15	55	36	91	2	7	2	11	324
25	26	28	50	49	99	5	11	11	27	400
10	13	19	47	41	88	0	3	3	6	401
20	28	29	55	47	102	5	9	7	21	402
20	15	24	48	38	86	3	5	3	11	403
17	18	21	45	45	90	1	5	5	11	404
12	14	13	39	37	76	0	5	6	11	405
13	18	21	53	41	94	3	6	8	17	406
11	13	8	33	34	67	0	3	3	6	407
8	10	21	48	42	90	2	7	6	15	408
15	19	26	52	42	94	0	7	5	12	409
20	19	23	51	44	95	2	8	8	18	410
17	22	24	51	47	98	4	8	6	18	411
15	18	24	46	44	90	3	9	7	19	412
15	15	19	54	47	91	2	8	5	15	413
15	18	18	55	41	96	2	6	5	13	414
15	17	25	58	45	103	6	12	10	28	415
25	20	27	56	48	104	2	6	8	16	416
12	19	26	39	35	74	3	7	5	15	417
12	8	16	48	38	86	4	10	5	19	418
13	15	13	51	24	75	0	4	4	8	419
15	18	22	52	46	98	4	6	7	17	420
17	17	26	54	44	98	6	11	8	25	421
19	21	26	43	41	84	4	7	4	15	422
21	20	26	54	46	100	4	6	4	14	423
15	20	23	51	36	87	4	7	5	16	424
12	9	17	34	27	61	3	4	3	10	501
15	21	17	39	39	78	4	6	4	14	502
20	20	29	53	46	99	4	11	10	25	503
11	12	17	49	35	84	3	5	2	10	504
19	21	21	54	42	96	3	6	6	15	505
16	23	23	49	43	92	5	10	6	21	506
14	23	17	43	39	82	3	6	4	13	507
13	12	16	44	37	81	3	4	4	11	508
16	19	21	39	39	78	4	9	5	18	509

EXPLORING MODERN MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
15	17	19	45	41	86	4	6	6	16	510
14	22	16	50	37	87	1	7	2	10	511
9	17	19	45	35	80	3	5	3	11	512
15	18	24	55	42	97	0	7	2	9	513
21	24	30	56	42	98	5	9	7	21	514
17	18	24	48	39	87	3	5	5	13	515
18	26	23	55	41	96	2	8	6	16	516
18	21	21	43	40	83	4	8	8	20	517
16	16	13	41	39	80	5	5	6	16	518
17	18	25	43	35	78	4	7	7	18	519
11	18	22	51	39	90	2	9	5	16	520
15	21	28	53	37	90	1	7	7	15	521
17	25	23	59	47	106	5	12	8	25	522
20	17	21	48	46	94	4	7	5	16	523
27	16	13	42	32	74	6	7	5	18	524
17	21	24	44	44	88	4	7	4	15	525
28	27	33	60	49	109	6	12	11	29	526
12	16	23	30	18	48	4	6	5	15	527
13	21	21	57	44	101	5	8	7	20	528
12	19	20	55	37	92	3	7	5	15	529
11	10	13	39	38	77	1	5	6	12	530
15	19	19	56	39	95	2	6	4	12	531
11	11	16	38	35	73	3	2	4	9	532
15	22	25	39	40	79	6	8	5	19	533
18	27	21	51	47	98	3	10	5	18	534
16	16	18	44	36	80	1	5	6	12	535
15	22	24	55	38	95	5	8	5	18	536
16	27	16	52	40	92	4	7	4	15	537
18	22	24	53	44	97	3	11	9	23	538
20	19	21	42	40	82	2	5	1	8	539
20	14	21	39	39	78	5	7	6	18	540
14	19	23	36	39	75	2	7	6	15	541
22	23	23	51	45	96	3	8	8	19	542
14	20	19	51	38	89	4	5	6	15	543
11	13	17	47	30	77	1	5	5	11	544
16	17	15	46	48	94	1	7	5	13	545
14	16	29	57	46	103	4	9	4	17	546
15	23	19	47	37	84	4	6	7	17	547
14	14	21	38	39	77	3	7	10	20	548
15	18	20	50	43	93	3	12	5	20	549
14	18	20	46	37	83	6	9	7	22	550
22	27	31	58	48	106	5	10	9	24	551
15	17	27	22	10	32	4	8	6	18	552
17	20	17	52	42	94	3	10	8	21	553
19	24	17	50	40	90	4	7	4	15	554

MATHEMATICS FOR CANADIAN STUDENTS

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
12	13	23	53	42	95	3	7	6	17	600
14	20	23	52	48	100	5	11	9	25	601
16	16	23	43	37	80	4	6	5	11	602
13	15	21	50	43	93	0	7	6	13	603
11	12	13	32	34	66	1	2	1	4	604
14	15	21	56	38	94	1	8	5	14	605
21	20	23	51	45	96	3	10	7	20	606
9	20	26	52	44	96	2	7	5	14	607
14	12	19	55	36	91	2	8	6	16	608
16	19	25	57	48	105	6	12	9	27	609
18	25	25	57	45	102	5	10	7	22	610
17	23	27	57	45	102	4	9	8	21	611
15	20	19	45	33	78	0	4	3	7	612
20	23	30	57	47	104	6	11	10	27	613
21	28	30	55	50	105	6	12	12	30	614
11	19	25	54	37	91	3	6	9	18	615
14	15	19	50	42	92	1	5	4	10	616
12	20	27	51	48	99	4	11	7	22	617
21	17	24	53	35	88	1	6	3	10	618
19	20	27	49	41	90	3	7	3	13	619
19	19	22	52	45	97	4	9	7	20	620
13	21	26	53	48	101	4	11	7	22	621
9	12	18	49	43	92	2	7	7	16	622
12	16	11	40	37	77	0	2	4	6	623
24	28	27	56	49	105	5	11	8	24	624
24	25	27	53	47	100	5	11	8	24	625
7	13	22	56	38	94	2	6	7	15	626
17	15	20	54	39	93	0	3	3	6	627
11	11	20	52	37	89	4	7	3	14	628
12	18	23	48	42	90	3	10	5	18	629
16	18	20	52	38	90	2	5	6	13	630
10	8	10	41	25	66	0	1	2	3	631
15	18	20	44	37	81	1	3	3	7	632
8	15	15	40	35	75	1	4	4	9	633
14	19	24	54	45	99	3	8	6	17	634
5	9	11	44	19	63	2	4	4	10	635
3	9	7	42	27	69	1	5	4	10	636
10	11	8	35	26	61	0	2	2	4	637
21	24	28	44	46	90	2	6	3	11	638
23	27	30	55	49	104	4	11	8	23	639
7	12	11	50	29	79	1	3	4	8	640
24	32	26	57	50	107	2	10	5	17	641
16	15	18	59	40	99	0	7	6	13	642

MATHEMATICS FOR CANADIAN STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
8	15	16	48	33	81	1	4	3	8	643
7	10	15	46	31	77	0	6	1	7	644
27	23	30	59	47	106	4	11	6	21	700
17	16	21	57	43	100	1	9	6	16	701
7	8	9	48	22	70	2	3	3	8	702
15	18	22	50	46	96	2	9	6	17	703
10	15	7	42	32	74	1	3	2	6	704
9	14	16	40	36	76	2	3	4	9	705
15	14	19	50	42	92	1	4	2	7	706
19	19	21	52	43	95	2	10	3	15	707
14	18	18	54	40	94	2	9	5	16	708
18	25	18	47	35	82	3	7	5	15	709
19	25	16	50	38	88	3	6	7	16	710
9	12	11	34	16	50	2	3	4	9	711
7	17	13	32	36	68	2	4	4	10	712
12	12	12	51	25	76	1	7	4	12	713
9	13	13	33	32	65	2	2	4	8	714
8	12	16	42	38	80	0	3	5	8	715
7	19	17	56	41	91	2	7	6	15	716
19	24	23	57	50	107	4	12	9	25	717
20	23	20	49	44	93	1	7	5	13	718
14	15	22	57	48	105	4	6	8	18	719
13	18	19	54	41	95	3	8	7	18	720
17	20	28	56	46	102	4	9	6	19	721
23	27	29	57	49	106	4	10	11	25	722
7	6	14	46	34	80	1	3	7	11	723
16	24	26	49	46	95	6	11	7	24	724
17	24	26	55	43	98	2	10	8	20	725
25	30	30	54	49	103	5	12	8	25	726
11	13	17	50	47	97	3	6	6	15	727
12	16	14	47	44	91	2	4	5	11	728
19	27	24	52	46	98	0	7	6	13	729
18	21	23	50	44	94	2	7	2	11	730
13	19	21	46	44	90	3	7	5	15	731
9	13	17	48	34	82	1	4	4	9	732
13	16	20	49	28	77	1	4	6	11	733
13	17	22	55	43	98	4	11	9	24	734
17	25	21	48	43	91	2	8	5	15	735
21	29	33	53	47	100	3	10	8	21	736
9	8	15	51	39	90	2	6	3	11	737
20	30	25	50	45	95	3	10	9	22	738
17	29	27	51	46	97	4	10	5	k9	739
5	11	18	53	35	88	1	5	4	10	740
24	28	28	58	47	105	5	9	11	25	741

MATHEMATICS FOR CANDIAN STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES				SPS SCORES			
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
30	32	27	56	50	106	4	10	9	23	742
13	18	18	48	47	95	2	10	5	17	743
17	22	19	55	46	101	3	9	7	19	744
15	17	27	50	47	97	3	11	11	25	745
20	25	26	53	48	101	2	6	4	12	746
15	19	16	51	42	93	1	9	7	17	747
20	26	22	53	46	99	2	3	5	10	748
17	25	26	57	49	106	2	7	5	14	749
13	21	23	51	47	98	0	7	4	11	750
10	15	17	46	35	81	1	3	2	6	751
9	12	10	46	17	63	2	3	6	11	800
21	24	25	46	48	94	5	8	7	20	801
13	25	26	41	48	89	3	8	7	18	802
15	22	24	46	47	93	0	9	3	12	803
15	23	17	40	40	80	1	8	4	13	804
15	16	25	39	40	79	3	5	3	11	805
7	11	15	47	29	76	1	4	3	8	806
11	16	14	51	33	84	0	5	6	11	807
10	17	25	51	45	96	2	8	6	16	808
4	20	25	37	44	81	3	5	8	16	809
13	16	21	44	34	78	0	5	4	9	810
17	15	20	37	40	77	1	4	3	8	811
14	17	19	41	44	85	1	6	5	12	812
6	18	17	47	34	81	0	7	4	11	813
14	18	23	50	32	82	2	3	1	6	814
14	21	28	47	43	90	2	7	6	15	815
19	18	27	48	46	94	4	10	8	22	816
8	20	10	34	34	68	1	5	3	9	817
16	20	23	46	39	85	3	7	5	15	818
13	19	25	44	46	90	2	6	4	12	819
19	18	24	54	45	99	2	5	5	12	820
14	15	28	36	40	76	2	7	1	10	821
15	17	20	44	34	78	1	3	4	8	822
9	25	27	39	41	80	1	6	3	10	823
18	21	28	50	39	89	1	7	4	12	824
25	31	34	53	49	102	4	9	10	23	825
22	15	26	50	42	92	2	6	4	12	826
10	15	25	32	39	71	1	8	5	14	827
8	8	13	28	29	57	2	3	2	7	828
13	10	10	37	33	70	1	2	2	5	829
21	22	28	57	48	105	2	11	10	23	830
20	16	24	48	41	89	3	8	6	17	831
13	17	20	43	44	87	2	7	4	13	832
14	21	22	54	40	94	2	8	6	16	833
9	19	28	53	44	97	3	5	7	15	834

MATHEMATICS FOR CANADIAN STUDENTS (cont'd)

	IOWA APS SCORES			SCAT SCORES			SPS SCORES			
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
14	24	24	46	47	93	2	5	5	12	835
11	9	19	39	37	76	2	6	2	10	836
17	17	20	56	36	92	1	7	7	15	837
10	22	25	39	40	79	1	8	3	12	838
13	19	17	38	44	82	2	7	4	13	839
18	25	25	54	43	97	3	6	4	13	840
12	17	22	45	41	86	3	7	4	14	841

SEEING THROUGH MATHEMATICS - EXPLORING MODERN MATHEMATICS STUDENTS

	IOWA APS SCORES			SCAT SCORES			SPS SCORES			
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
20	26	29	58	47	105	6	12	10	28	900
14	19	21	59	43	102	4	9	8	21	901
16	19	22	55	47	102	2	7	7	16	902
12	21	23	51	37	88	3	9	9	21	903
12	21	24	31	34	65	2	7	1	10	904
17	22	23	55	43	98	3	9	7	19	905
10	21	18	46	37	83	3	5	6	14	906
7	9	19	49	42	91	2	7	7	16	907
9	12	8	35	31	66	2	5	7	14	908
6	15	11	40	33	73	1	5	1	7	909
10	14	21	56	42	98	1	10	8	19	910
8	11	11	47	26	73	1	3	1	5	911
19	21	20	57	42	99	2	8	6	16	912
13	19	21	54	41	95	1	7	7	15	913
10	20	14	48	38	86	1	5	2	8	914
9	9	15	23	25	48	0	2	4	6	915
10	15	16	56	37	93	0	7	6	13	916
9	13	20	48	36	84	1	0	4	5	917
10	12	9	46	30	76	0	5	3	8	918
11	10	15	52	40	92	3	7	6	16	191
14	18	16	41	32	73	1	6	3	10	920
12	9	12	42	30	72	0	5	2	7	921
5	5	14	42	18	60	1	3	3	7	922
6	6	10	38	30	68	1	4	5	10	923
16	19	22	49	44	93	5	8	7	20	924
17	17	16	53	46	99	1	11	8	20	950
28	26	26	58	50	108	5	12	11	28	951
9	19	24	52	44	96	4	10	9	23	952
22	25	30	52	44	96	5	9	10	24	953
19	16	27	52	41	93	1	7	2	10	954

SEEING THROUGH MATHEMATICS - EXPLORING MODERN MATHEMATICS STUDENTS (cont'd)

IOWA APS SCORES			SCAT SCORES			SPS SCORES				
Sept. 1963	Mar. 1964	May 1965	V	Q	T	Q	P	L	T	I.D.
16	22	24	54	49	103	6	9	5	20	955
21	27	30	55	45	100	4	9	10	23	956
16	14	18	57	42	99	3	7	6	16	957
18	20	27	57	46	103	5	12	10	27	958
19	19	24	56	47	103	5	8	6	19	959
12	13	22	55	42	97	1	7	10	18	960
18	23	21	57	46	103	3	8	5	16	961
22	21	28	56	50	106	5	12	12	29	962
23	21	25	48	48	96	3	9	7	19	963
17	20	26	61	46	107	2	9	5	16	964
18	20	27	51	50	101	4	10	8	22	965
17	19	24	52	42	94	6	11	9	26	966
17	23	24	56	46	102	2	9	9	20	967
17	17	25	55	41	96	3	7	7	17	968
20	22	23	57	48	105	5	9	10	24	969
22	24	30	54	46	100	1	8	6	15	970
16	16	21	50	43	93	3	5	6	14	971
20	13	22	50	41	91	1	6	4	11	972
24	19	22	44	46	90	2	8	3	13	973
18	12	14	55	49	104	2	5	9	16	974
6	9	22	43	41	84	2	7	7	16	975
17	21	30	54	47	101	3	10	10	23	976

APPENDIX D

CHI SQUARE TEST ON CELL FREQUENCIES

SCAT Ability Level	GROUP				TOTAL
	STM	EMM	TRAD	STM-EMM	
H	66 (66)	28 (35)	45 (47)	27 (18)	166
A	62 (66)	43 (35)	48 (47)	12 (17)	165
L	69 (65)	33 (34)	46 (45)	13 (17)	161
Total	197	104	139	52	492

Numbers in parentheses are expected cell frequencies.

$O - E$	$(O - E)^2$	$\frac{(O - E)^2}{E}$
0	0	0.
-4	16	.24
4	16	.24
-7	49	1.40
8	64	1.83
-1	1	.03
-2	4	.08
1	1	.02
1	1	.02
9	81	4.50
-5	25	1.47
-4	16	.94

df = (R-1)(C-1) = 6 $\chi^2 =$ 10.77

$\chi^2_{.05}$ (6df) = 12.59 $\chi^2_{.01}$ (6df) = 16.81

SUMMARY OF ANALYSIS OF VARIANCE OF TEST SCORES ATTAINED DURING INITIAL TESTING

Analysis of Variance, September 1963, IOWA APS Scores

Source	SS	MS	DF	F
Groups	0.13826270E 03	0.46087565E 02	3	1.98
Error	0.11384055E 05	0.23327981E 02	488	

Analysis of Variance, March 1964, IOWA APS Scores

Source	SS	MS	DF	F
Groups	0.10584375E 03	0.35381250E 02	3	1.26
Error	0.13614662E 05	0.27898898E 02	488	

Analysis of Variance, September 1963, SCAT V Scores

Source	SS	MS	DF	F
Groups	0.14168750E 03	0.47229166E 02	3	1.03
Error	0.22360031E 05	0.45819736E 02	488	

Analysis of Variance, September 1963, SCAT Q Scores

Source	SS	MS	DF	F
Groups	0.56726562E 02	0.18908854E 02	3	0.44
Error	0.21167547E 05	0.43376121E 02	488	

Analysis of Variance, September 1963, SCAT T Scores

Source	SS	MS	DF	F
Groups	0.31931250E 03	0.10643750E 03	3	0.79
Error	0.66054875E 05	0.135335835E 03	488	

Critical F-ratio $F_{.01}(3,488) = 3.78$

"t" TEST STATISTIC

$$t = \frac{A_1 - A_2}{\frac{2MS_{w.cell}}{n_h q}}$$

$$df = N - pq$$

$\bar{A}_1$ is the mean of means for row 1

$\bar{A}_2$ is the mean of means for row 2

$MS_{w.\ cell}$ is the pooled within cell variance

n_h is the harmonic mean of the cell frequencies

N is the total number of scores involved

p is the number of rows

q is the number of colums

df is the number of degrees of freedom.

SUMMARY OF ANALYSIS OF VARIANCE OF TEST SCORES ATTAINED DURING THE FINAL TESTING

Analysis of Variance, May 1965, IOWA APS Scores

Source	SS	MS	DF	F
Groups	0.33324219E 02	0.11108073E 02	3	0.34
Error	0.15801723E 05	0.32380579E 02	488	

Analysis of Variance, May 1965, SPS Scores

Source	SS	MS	DF	F
Groups	0.41845703E 03	0.13948568E 03	3	4.46
Error	0.15274227E 05	0.31299644E 02	488	

Analysis of Variance, May 1965, SPS (Linear Section) Scores

Source	SS	MS	DF	F
Groups	0.91799437E 02	0.30599812E 02	3	5.22
Error	0.28620299E 02	0.58648153E 01	488	

Analysis of Variance, May 1965, SPS (Proportional Section) Scores

Source	SS	MS	DF	F
Groups	0.30206787E 02	0.10068929E 02	3	1.61
Error	0.30457607E 04	0.62413129E 01	488	

Analysis of Variance, May 1965, SPS (Quadratic Section) Scores

Source	SS	MS	DF	F
Groups	0.47655609E 02	0.15885203E 02	3	6.43
Error	0.12050436E 04	0.24693516E 01	488	

Critical F-ratio $F_{.01}(3,488) = 3.78$

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